



STA301

Final-Term (Solved)

ABSTRACT

This comprehensive collection of notes is accurately crafted to empower students to excel academically, ensuring they achieve a minimum of 80% marks in their examinations. The content is organized with clarity and precision, focusing on key concepts, critical analyses, and practical applications tailored to the syllabus. These notes serve as a reliable resource for both thorough preparation and last-minute revision. Designed to inspire confidence and mastery, this guide is an essential tool for students striving for academic excellence.

Maha Malik

Q. Random Sampling

Definition

Random sampling is the method in which every member of the population has an equal chance of being selected. It reduces bias and helps the sample represent the population. The STA301 handout describes simple random sampling as selecting units by chance (lottery method or random numbers).

Advantages

- Equal chance for every member
- Less bias
- Simple to understand
- Representative sample
- Better statistical inference

Types

- Simple Random Sampling
- Stratified Sampling
- Systematic Sampling
- Cluster Sampling
- Multistage Sampling

Q. PDF (Probability Density Function)

For a continuous random variable,

- $f(x) \geq 0$
- Total area under curve = 1

Probability is calculated as

$$P(a < X < b) = \int_a^b f(x) dx$$

Remember:

A PDF gives probability over an interval, not at a single point.

Q. What is the effect of degrees of freedom on Chi-square distribution?

Answer

As the degrees of freedom increase:

- The distribution becomes less skewed.
- It becomes more symmetric.
- It spreads to the right.
- Its mean equals its degrees of freedom.

Q. What is the singular of "Strata"?

Answer

Stratum

Q. Write any two types of non-random sampling.

Answer

- Convenience Sampling
- Judgment Sampling

Q. Probability

Formula

$$P(E) = \frac{\text{Favourable Outcomes}}{\text{Total Outcomes}}$$

Example

A die is rolled.

Probability of getting 4

$$P(4) = \frac{1}{6}$$

Probability of even number

$$P = \frac{3}{6} = \frac{1}{2}$$

Q. Which data collection method is used when an investigator personally collects information from individuals?

Answer

Personal Interview (Personal Investigation Method)

Q. Find Mean and Variance

Mean

$$\mu = \frac{\sum X}{n}$$

Variance

$$\sigma^2 = \frac{\sum (X - \mu)^2}{n}$$

Example

Data:

2,4,6

Mean

$$\mu = \frac{2 + 4 + 6}{3} = 4$$

Variance

$$= \frac{(2 - 4)^2 + (4 - 4)^2 + (6 - 4)^2}{3} = \frac{4 + 0 + 4}{3} = \frac{8}{3} = 2.67$$

Mean = 4

Variance = 2.67

Q. Which distribution is used to determine the proportion of smokers and non-smokers?

Answer

Binomial Distribution

Reason:

Each person belongs to only one of two categories:

- Smoker
- Non-smoker

Q. Name the measures of dispersion that are NOT based on all observations.

Answer

- Range
- Quartile Deviation (Semi-Interquartile Range)

Q. When do we use a two-tailed test?

Answer

A two-tailed test is used when the alternative hypothesis checks for **either an increase or a decrease** (any difference).

Q. Integral of Density Function

Given

$$f(x) = \frac{2}{5}x^2$$

Integrate

$$\int \frac{2}{5}x^2 dx$$

Solution

$$= \frac{2}{5} \cdot \frac{x^3}{3} = \frac{2x^3}{15} + C$$

Answer

$$\boxed{\frac{2x^3}{15} + C}$$

If limits (0 to a, etc.) are given, substitute upper limit minus lower limit.

Q. Test Hypothesis (Numerical)

Example

Compilation

Example

Given

- $n = 36$
- $\bar{X} = 52$
- $\mu = 50$
- $\sigma = 6$
- $\alpha = 0.05$

Step 1

$$H_0 : \mu = 50$$

$$H_1 : \mu \neq 50$$

Step 2

$$\begin{aligned} Z &= \frac{52 - 50}{6/\sqrt{36}} \\ &= \frac{2}{1} = 2 \end{aligned}$$

Critical value

$$\pm 1.96$$

Since

$$2 > 1.96$$

Reject H_0 .

Conclusion: There is sufficient evidence that the population mean is different from 50.

Q. State the null and alternative hypotheses if a new machine is expected to reduce variability.

Given:

- Old population standard deviation: $\sigma_0 = 15.2$ g
- Sample size: $n = 10$

Answer

Since the manager wants to know whether the **new machine reduces variability**, this is a **left-tailed chi-square test**.

Null Hypothesis (H_0):

$$\sigma^2 \geq (15.2)^2$$

(or equivalently, $\sigma^2 = (15.2)^2$ is included in H_0)

Meaning: The new machine has the same or greater variability.

Alternative Hypothesis (H_1):

$$\sigma^2 < (15.2)^2$$

Meaning: The new machine has lower variability (better performance).

Test Statistic

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

Here,

$$\chi^2 = \frac{9s^2}{231.04}$$

Degrees of freedom:

$$df = n - 1 = 9$$

Q. What is the probability that a 5-card hand contains exactly two aces?

Answer

Use the **Hypergeometric Distribution**.

Given:

- Total cards = 52
- Number of aces = 4
- Cards drawn = 5
- Required aces = 2

Formula:

$$P(X = x) = \frac{\binom{K}{x} \binom{N-K}{n-x}}{\binom{N}{n}}$$

Substitute:

$$P(X = 2) = \frac{\binom{4}{2} \binom{48}{3}}{\binom{52}{5}}$$

Q. Find Mean

Formula (Ungrouped Data)

$$\bar{X} = \frac{\sum X}{n}$$

Formula (Frequency Distribution)

$$\bar{X} = \frac{\sum fX}{\sum f}$$

Example

Data:

2, 4, 6, 8, 10

$$\bar{X} = \frac{2 + 4 + 6 + 8 + 10}{5} = \frac{30}{5} = 6$$

Answer = 6

Q. What is the probability that a 5-card hand contains exactly one ace?

Answer

Use

$$P(X = 1) = \frac{{}^4_1 {}^{48}_4}{{}^{52}_5}$$

Final Answer:

0.29947

Q. Given:

Compilation

- $n = 22$
- $\sum(d - \bar{d})^2 = 270$
- $\bar{d} = 4$
- $\alpha = 0.05$

Test

$$H_0 : \mu_d = 0$$

against

$$H_1 : \mu_d \neq 0$$

Answer

Standard deviation

$$s_d = 3.586$$

Test statistic

$$t = \frac{\bar{d} - \mu_0}{s_d / \sqrt{n}}$$

Result

$$t = 5.23$$

Degrees of freedom

$$df = 21$$

Critical value

$$t_{0.025, 21} = 2.08$$

Since

$$5.23 > 2.08$$

Reject H_0 .

Conclusion:

There is significant evidence that

$$\mu_d \neq 0$$

Q. For a sample size $n = 15$ and $\alpha = 0.10$, find

Compilation

$$t_{\alpha/2}$$

Answer

Degrees of freedom

$$v = n - 1 = 14$$

$$\alpha/2 = 0.05$$

From the t-table,

$$t_{0.05,14} = 1.761$$

Q. If

Mean = 20

Standard deviation = 2

Find $\mu \pm 2\sigma$ limits.

Lower limit

$$20 - 2(2) = 16$$

Upper limit

$$20 + 2(2) = 24$$

Therefore

$$16 \leq X \leq 24$$

According to the empirical rule,

95% of observations lie within these limits.

Q. If X follows a Poisson distribution with parameter $\lambda = 3$, find $P(X = 1)$.

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Formula

$$P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

Substitute

$$P(X = 1) = \frac{e^{-3} (3)^1}{1!}$$

Since

$$e^{-3} = 0.0498$$

Final answer

$$0.1494$$

Q. Given:

$$x = 341$$

$$n = 634$$

$$p_0 = 0.50$$

Find the Z-test statistic for proportion.

Answer

Since

$$x > np_0$$

use continuity correction.

Final answer

$$Z = 1.87$$

Q. Given:

Compilation

$$\mu = 5$$

$$\bar{x} = 7$$

$$s = 2$$

$$t = 3$$

Find sample size.

Answer

Using

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

Result

$$n = 9$$

Q. Given:

$$X \sim N(50, 25)$$

Find

$$P(X > 54)$$

Standard deviation

$$\sigma = 5$$

Z-score

$$z = \frac{54 - 50}{5} = 0.8$$

From Z-table

$$P(Z < 0.8) = 0.7881$$

Therefore

$$P(Z > 0.8) = 1 - 0.7881 = 0.2119$$

Q. Describe Poisson distribution as the limiting form of Binomial distribution.

Poisson distribution is obtained from the Binomial distribution when:

1. Number of trials becomes very large

$$n \rightarrow \infty$$

2. Probability of success becomes very small

$$p \rightarrow 0$$

3. Mean remains finite

$$\lambda = np$$

Under these conditions,

Binomial distribution approaches the Poisson distribution.

Q. Check whether function is valid

Usually a density function is valid if

Condition 1

$$f(x) \geq 0$$

Condition 2

$$\int_a^b f(x) dx = 1$$

If both conditions are satisfied,

The function is VALID.

Otherwise,

NOT VALID.

Q. Define Acceptance Region.

Answer

The acceptance region is the range of values of the test statistic for which we **do not reject the null hypothesis**.

Example:

For a two-tailed Z-test at 5% significance,

Critical values are

$$-1.96, +1.96$$

Accept H_0 if

$$-1.96 \leq Z \leq 1.96$$

Reject H_0 otherwise.

Q. Find the sampling error if

Population mean = 15.785

Sample mean = 15.600

Answer

Sampling Error

$$= 15.785 - 15.600$$

$$= 0.185$$

Q. Alternative Hypothesis

Alternative hypothesis is written as

Right-tailed

$$H_1 : \mu > \mu_0$$

Left-tailed

$$H_1 : \mu < \mu_0$$

Two-tailed

$$H_1 : \mu \neq \mu_0$$

Decision Rule

- Reject H_0 if Test Statistic falls in rejection region.
- Otherwise accept (or fail to reject) H_0 .

Q. A coin is tossed 900 times and heads appear 490 times.

Write null and alternative hypotheses to test whether the coin is unbiased.

Answer

Null hypothesis

$$H_0 : p = 0.5$$

Coin is unbiased.

Alternative hypothesis

$$H_1 : p \neq 0.5$$

Coin is biased.

Q. Find the probability of drawing a white ball from a bag containing

- 4 red
- 8 black
- 3 white

Answer

Total balls

15

White balls

3

Probability

$$\frac{3}{15} = 0.20$$

Q. Write the properties of the sampling distribution of $\hat{P}$ when sampling is performed without replacement

Compilation

Answer

1.

$$E(\hat{P}) = P$$

2.

$$SE(\hat{P}) = \sqrt{\frac{PQ}{n} \times \frac{N-n}{N-1}}$$

where

- P = population proportion
- $Q = 1 - P$
- n = sample size
- N = population size

3.

If sample size is large,

the sampling distribution of $\hat{P}$ is approximately normal.

Q. Continuous Integration Question

Probability is

$$P(a < X < b) = \int_a^b f(x) dx$$

Example

$$f(x) = 2x, \quad 0 < x < 1$$

Find

$$P(0 < X < 0.5)$$

Solution

$$\begin{aligned} &= \int_0^{0.5} 2x \, dx \\ &= x^2 \Big|_0^{0.5} \\ &= 0.25 \end{aligned}$$

Q. Given:

$$x = 255$$

$$n = 500$$

$$p_0 = 0.60$$

Find Z-test statistic.

Answer

First,

First,

$$\hat{P} = \frac{255}{500} = 0.51$$

Final answer

$$Z = -4.1$$

Q. Population = {1,2,3,5,6}; sample size = 3 (without replacement).

Find the sampling distribution of the sample proportion of even numbers.

Answer

Population proportion

$$P = \frac{2}{5}$$

Total samples

$$\binom{5}{3} = 10$$

Possible values of $\hat{P}$:

- 0
- $1/3$
- $2/3$

Probabilities:

Sample Proportion

Probability

0

$1/10$

$1/3$

$6/10$

$2/3$

$3/10$

Q. Hypothesis Testing

Compilation

Steps

1. Write Null Hypothesis H_0
2. Write Alternative Hypothesis H_1
3. Select significance level ($\alpha = 0.05$)
4. Compute test statistic (Z or t)
5. Find critical value or p-value
6. Compare values
7. Accept or Reject H_0
8. Write conclusion

Z-test Formula

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

Q. Write the testing procedure for the Chi-square Goodness-of-Fit Test.

1. State H_0 and H_1 .
2. Choose significance level (α).
3. Calculate expected frequencies.
4. Compute

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

5. Find degrees of freedom.
6. Compare calculated χ^2 with the table value.
7. If calculated χ^2 is greater than the critical value, reject H_0 ; otherwise, do not reject H_0 .

Q. Given:

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- $n = 634$
- $\hat{P} = 0.459$
- $Z_{\alpha/2} = 2.58$

Find the 99% confidence interval for the population proportion.

Answer

Standard error

$$SE = 0.0198$$

Margin of error

$$ME = 2.58(0.0198) = 0.0511$$

Lower limit

$$0.459 - 0.0511 = 0.4079$$

Upper limit

$$0.459 + 0.0511 = 0.5101$$

99% Confidence Interval

$$(0.4079, 0.5101)$$

Important Formulas for STA301 Exam

Topic	Formula
Mean	$\bar{X} = \frac{\sum X}{n}$
Mean (Frequency)	$\frac{\sum fX}{\sum f}$
Variance	$\sigma^2 = \frac{\sum (X - \mu)^2}{n}$
Probability	$\frac{\text{Favourable}}{\text{Total}}$
Continuous Probability	$\int_a^b f(x) dx$
PDF Conditions	$f(x) \geq 0, \int f(x) dx = 1$
Z-Test	$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$
Sample Mean	$\bar{X} = \frac{\sum X}{n}$

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Compilation

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