Final term Notes

Mth304

Lecture # 23

Moment of a force about a point

- **Introduction to Moments:-**

The Tendency of a force to rotate a rigid body about any defined axis is called the Moment of the force about the axis.

The moment, M, of a force about a point provides a measure of the tendency for rotation (sometimes called a torque).

$$M = F * d$$

- **Moment Caused by a force:-**

* The Moment of force (F) about an x-axis through point (A) or for short, the Moment of

of F about A is the product of the Magnitude of the force and the perpendicular distance between point (A) and the line of action of force (F)

$\therefore M_A = Fd$.

UNITS of Moment:-

* N-m in the SI System.
* ft-lbs or In-lbs is the US customary System.

Properties of a Moment:-

* Moments not ~~not~~ only have a magnitude, they also have a Sense to them.

* The Sense of a moment is clockwise or Counter-Clockwise depending on which way

it will tend to make the object rotate.

* The Sense of a Moment is defined by the direction it is acting on the Axis and Can be found using Right Hand Rule.

* In the 2-D Case, the magnitude of the moment is
$M_O = Fd$.

In 2-D, the direction of M_O is either Clockwise or Counter-Clockwise, depending on the tendency for rotation.

Lecture # 24

Varignons Theorms.

*** Cross product of two vectors:-**

Given two non-zero vectors $a = \langle a_1, a_2, a_3 \rangle$ and $b = \langle b_1; b_2, b_3 \rangle$, It is very useful to be able to find a non zero vector c that is perpendicular to both a and b.

if $c = \langle c_1, c_2, c_3 \rangle$ is such a vector, then $a \cdot c = 0$ and $b \cdot c = 0$ and so

(1) $a_1 c_1 + a_2 c_2 + a_3 c_3 = 0$

(2) $b_1 c_1 + b_2 c_2 + b_3 c_3 = 0$

To eliminate c_3 we Multiply (1) by b_3 and (2) by a_3 and Subtract:

③ $(b_1 b_3 - a_3 b_1) c_1 + (a_2 b_3 - a_3 b_2) c_2 = 0$

Equation 3 has the form $pc_1 + qc_2 = 0$, for which an obvious solution $c_1 = q$ and $c_2 = -q$

So a solution of ③ is:-

$$c_1 = a_2 b_3 - a_3 b_2$$
$$c_2 = a_3 b_1 - a_1 b_3$$

if $a = \langle a_1, a_2, a_3 \rangle$ and $b = \langle b_1, b_2, b_3 \rangle$, then the Cross product of a and b is the vector.

$$a \times b = \langle a_2 b_3 - a_3 b_2 - a_3 b_2 \, a_3 b_1 - a_1 b_3 \, a_1 b_2 - a_2 b_1 \rangle$$

Notice that the Cross product $a \times b$ of two vectors a and b, unlike the dot product, is a vector.

for this reason. it is also called the vector product.

Note that axb is defined only when a and b are there-diemensional vectors.

Also watch video Lecture clear for concepts from Examples of Determinet.

* Vaarignon's Theorem:-

"The moment of a force about any point is equal to the Sum of the moments of the Components of the force about the Same point."

The moment of R about point O is Rd. However, if d is more difficult to determine than P and q, we can resolve R into the Components, P and Q, and the Moment as.

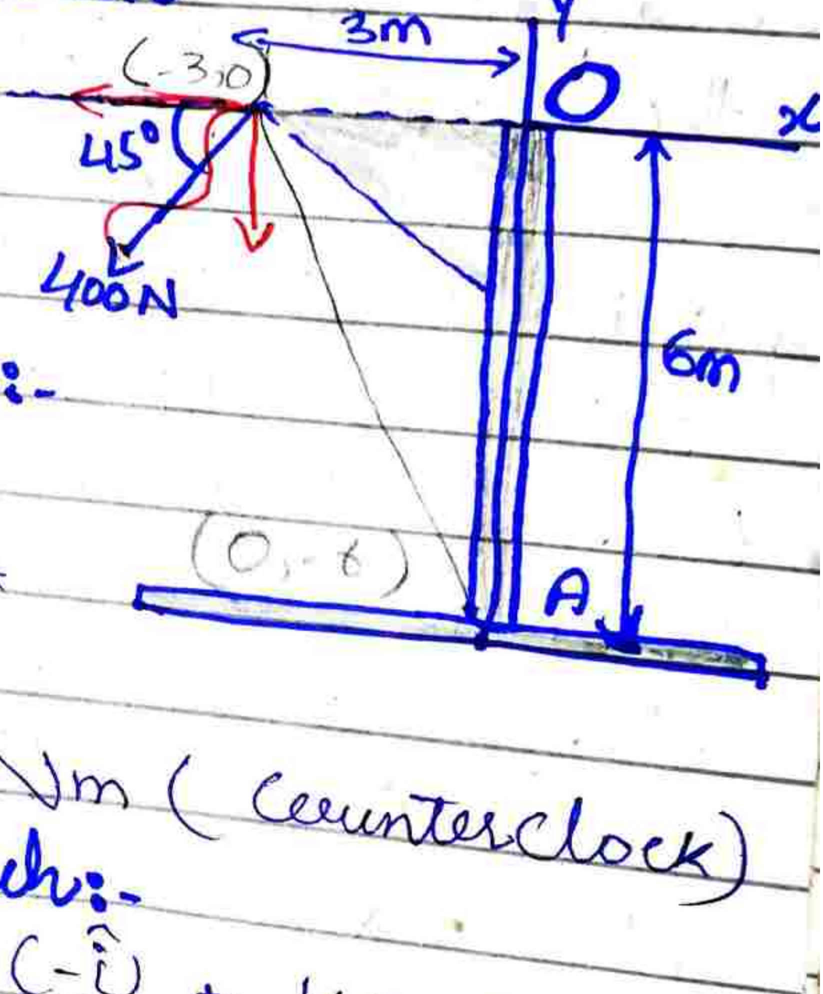
$$M_o = Rd = -pP + qR$$

$$\left.\begin{aligned} M_o &= r \times R \\ r \times R &= r \times (P + Q) \\ M_o = r \times R &= r \times P + r \times Q \end{aligned}\right] \text{ vector Process.}$$

where we take the Clock-wise moment sense to be positive.

Problem

for the figure shown below, find moment of 400N force about point A using:-
1- Scalar approach
2- vector approach



1- Scalar approach:-

$M_A = 400 \cos 45° \times 6 + 400 \sin 45° \times 3$

$M_A = 2545.2$ Nm (counterclock)

2- Vector approach:-

$\vec{F} = 400 \cos 45° (-\hat{i}) + 400 \sin 45° (-\hat{j})$

$= 282.8\hat{i} - 282.8\hat{j}$

$\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$

$= (-3 - 0)\hat{i} + (0-(-6))\hat{j}$

$= -3\hat{i} + 6\hat{j}$

$$M_{A} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 6 & 0 \\ -282.8 & -282.8 & 0 \end{vmatrix}$$

$= 2545.2\,\hat{k}$ Nm <u>Ans.</u>

Lecture # 25

Second Condition of Equilibrium:-

Net Moment:-

* The net moment is the sum of all the moments produced by all the forces.

- Remember to account for the ~~position~~ direction of the tendency for rotation:-
  - Counterclock wise moment are positive
  - Clockwise moment are negative.

Moment and Equilibrium:-

* First condition of equilibrium
  - The net external force must be zero

$$\sum \vec{F} = 0 \quad \text{or}$$

$$\sum \vec{F}_x = 0 ; \quad \sum \vec{F}_y = 0$$

  - This is a necessary, but not sufficient, condition to ensure that an object is in complete mechanical equilibrium.

* This is a statement of translational equilibrium.

* To ensure mechanical equilibrium you need to ensure rotational equilibrium as translational.

* The Second Condition of Equilibrium states:-

* The net moment must be zero

$$\sum M_i = 0$$

Total Clockwise Moments = Total Anti-clockwise Moments.

Moments of uniform rod about a point.

Rods:-

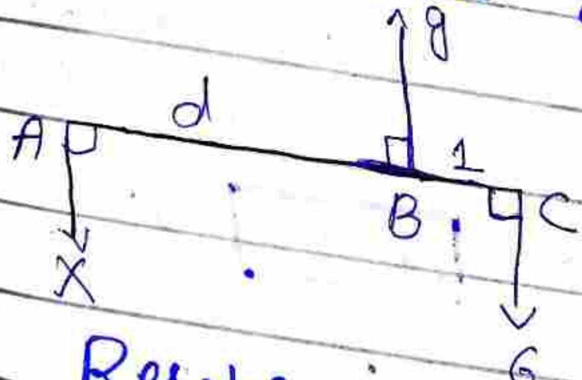
* if a rod is $\boxed{\text{uniform}}$ then its weight can be taken to act at the Mid-point of the rod.
* if a rod is $\boxed{\text{pivoted}}$ at a point, then there will be a reaction force at this point.
* if a rod is resting on <u>two or more supports</u>, then there will be reaction forces at these points.

* Light uniform rods:-

A light rod may have a mass so small compared to the masses acting on it, that its own mass can be ignored.

Example:-

1- The light rod shown below is in equilibrium. Calculate the size of the force X and value of d.



Resolving forces vertically:-

$X+6=9$

So, $X=3$

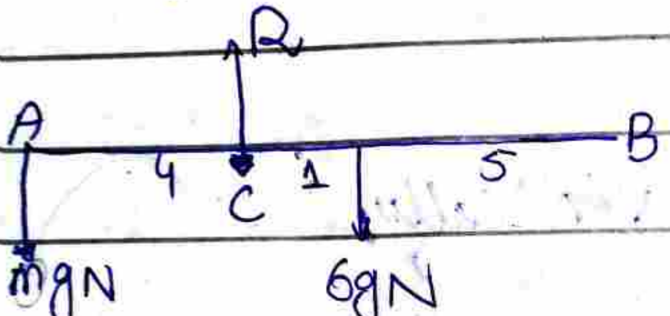
Taking moments about C:-

$9\times 1 = 3(d+1)$

$9-3 = 3d$

$d=2$

2. A uniform rod AB has mass of 6kg and is pivoted at C. The length of rod is 10m and distance AC = 4m. Calculate the mass of the weight that must be attached at A to keep the rod in equilibrium.



Taking moments about C:

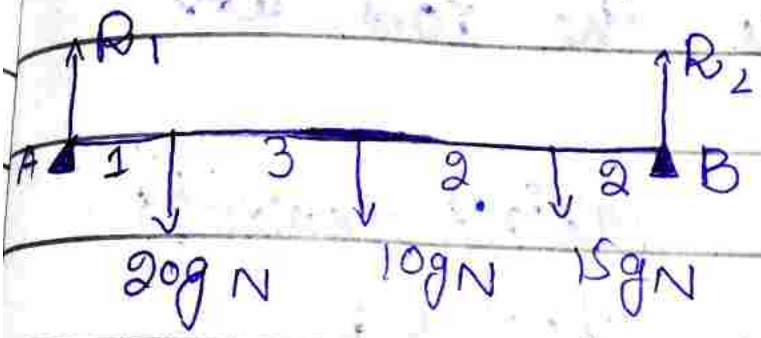
$m \times 4 = 6 \times 1$

$4m = 6$

$m = 1.5$

Therefore, the mass of the weight that must be attached at A is 1.5 kg.

3- A uniform beam AB of mass 10kg is 8m long and is resting on two supports at A & B. A mass of 20kg is placed on the beam 1m from A. A mass of 15kg is placed on the beam 2m from B. Calculate the reaction forces at A and B.



Resolving forces vertically:-

$$R_1 + R_2 = 45\,gN$$

Taking Moments about A:-

$$20 \times 1 + 10 \times 4 + 15 \times 6 = R_2 \times 8$$

$$150 = 8R_2$$

$$R_2 = 18.7gN$$

So, $R_1 = 45 - R_2$

$$R_1 = 45 - 18.75$$

$$R_1 = 26.25N.$$

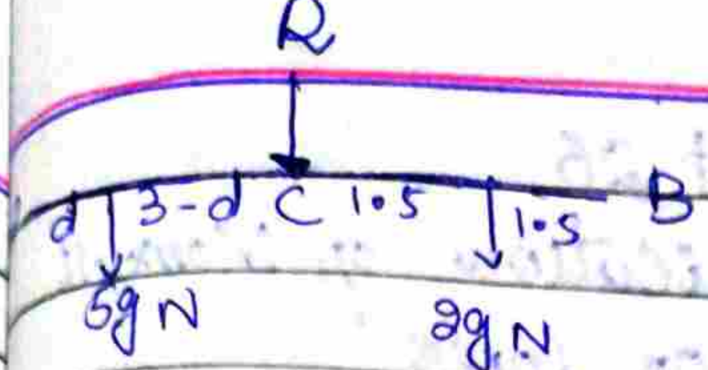
Lecture # 26
Application of Moment of force

• Moments of Non-uniform rod about a point

✓ The Centre of mass of a non-uniform rod is not necessarily at the rod's mid-point.

Example :-

1- A non-uniform beam AB of length 6m and mass 5kg rests on a pivot at the mid-point C. When a mass of 2kg rests on the beam 1.5m from B, the system is in equilibrium. Find the distance between A and the centre of mass of the beam



Taking Moments about C:-

$$5g \times (3-d) = 2g \times 1.5$$

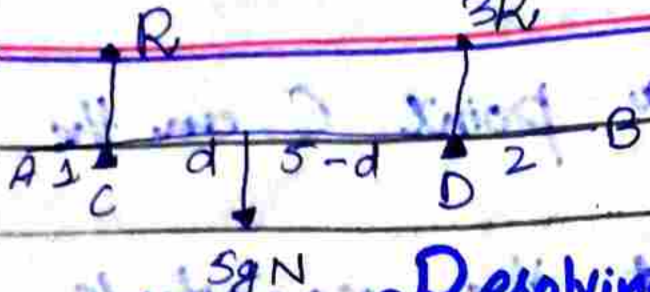
$$15 - 5d = 3$$

$$\text{So } 5d = 12$$

$$d = 2.5\text{m}$$

Therefore, the Centre of mass of the beam is 2.4m from A.

(2) A non-uniform rod AB has length 8m and mass 5kg. It is resting on two supports at C & D, where C is 1m from A and D is 2m from B. The reaction force at C is a third of that at D. Calculate the distance between A & the centre of mass of the rod.



Resolving vertically:-

$$4R = 5gN$$
$$R = 1.25gN$$

Taking moments about D:-

$$5R = 5g(5-d)$$
$$6.25g = g(25-5d)$$
$$6.25g = g(25-5d)$$
$$6.25 = 25-5d$$
$$5d = 18.75$$
$$\therefore d = 3.75m$$

So, the Centre of mass is 4.75m from A.

Example of Moment:-

1- A footbridge across a stream Consist of a tree trunk of length 6m and mass 100kg, supported at the ends A and B.

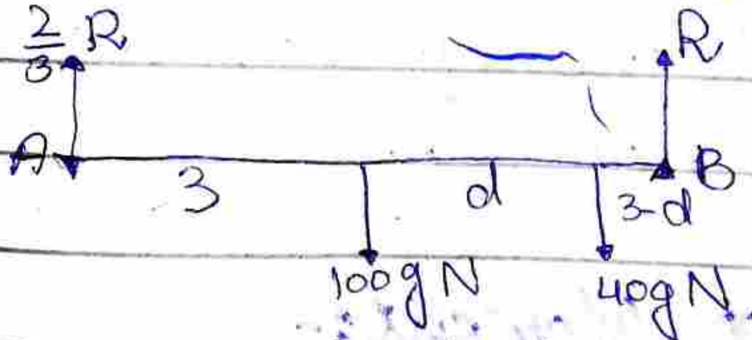
A Child of Mass 40kg is

Standing at a point C on the footbridge.

Given that the magnitude of the force exerted by the Support at A is two thids of the Magnitude of the force exerted by the Support at B, Calculate;

(a) the Magnitude, in Newtons of the force exerted by the Support at A,

(b) the distance AC.



(a) Resolving force vertically:-

$$\frac{2}{3}R + R = 100gN + 40gN$$

$$\frac{5}{3}R = 140gN$$

$$\therefore R = 84gN$$

force At A $= \frac{2}{3} \times \overset{28}{\cancel{84}}gN$

$$A = 56gN$$

(b) Taking moments about A:-

2^{nd} condition of equilibrium:- $(100\times3)+40(3+d)=84\times6$

$300+120+40d=504$

$40d=84$

$d=2.1\,m$

Therefore the distance AC is

$=d+3$

$=2.1+3=5.1m.$

Example:-

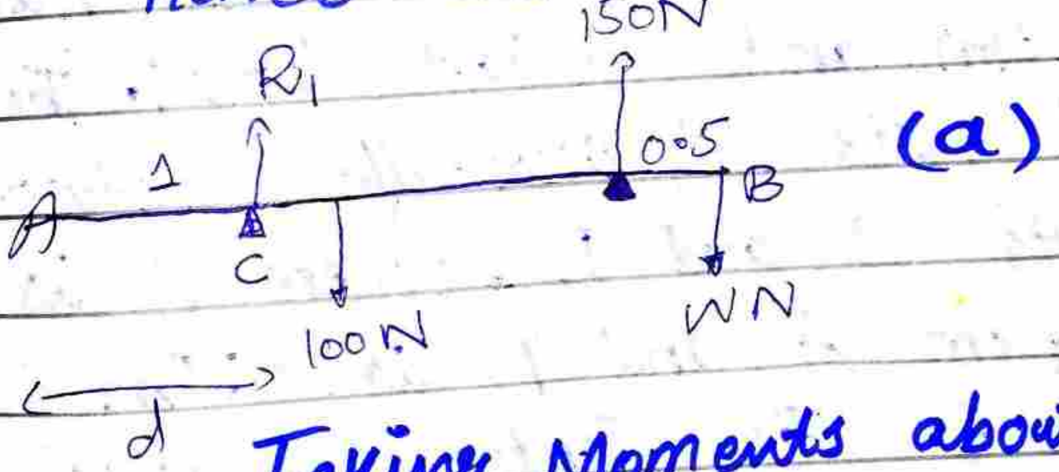
A non-uniform rod AB has length 4m and weight 100 N. The rod rests horizontally on two supports C and D, where C is 1m from A and D is 0.5m from B. The Centre of Mass of the rod is d m from A.

An object of weight W Newton is placed at B. The Magnitude of the reaction force on the rod at D is now 150 N.

(a) Show that $100d + 3W = 475$.

The object is now moved from B and placed at A. The rod remains in equilibrium and the magnitude of the reaction force at D is now 60 N.

(b) Write another equation connecting W & d, and Hence calculate W & d.

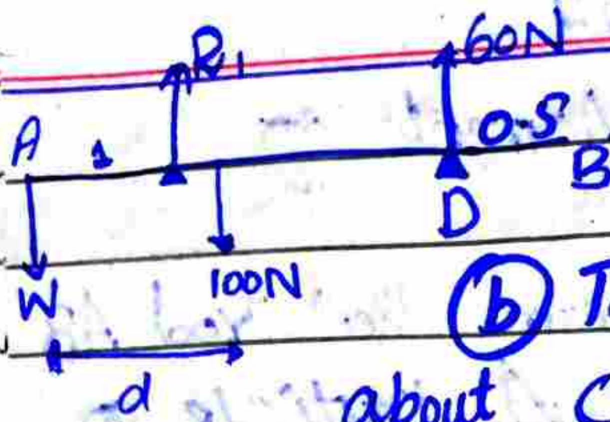


Taking Moments about C:

$100 \times (d-1) + 3W = 150 \times 2.5$

$100d - 100 + 3W = 375$

$100d + 3W = 475$



(b) Taking moments about C:

$$100 \times (d-1) = 1W + 60 \times 2.5$$

$$100d - 100 = W + 150$$

$$100d - W = 250$$

Solving simultaneously:-

$$\cancel{100d} + 3W = 475$$

$$\underline{\pm \cancel{100d} \mp W = \pm 250}$$

$$4W = 255$$

$$W = 56.25 \text{ N}$$

and $d = 3.06$ m by putting $W = 56.25$ in equation.

lecture # 27

Examples of Moment of forces

A heavy uniform steel rod AB has length 12m. A particle of weight 100N is placed at A and a particle of weight 200N is placed at B.

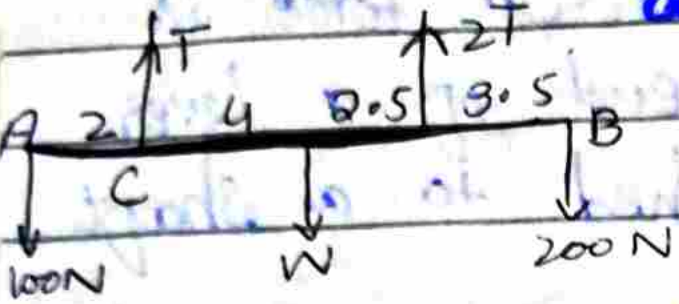
The rod is attached to the ceiling by two steel cables at C and D where C is 2m from A & D is 3.5m from B.

The tension in the cable at D is twice the tension in the cable at C.

(a) Calculate the Tension in the cable at C and state any assumptions that you have mde.

(b) Calculate the weight of the steel rod.

Solution :- (a) Taking moments about the Centre of the rod:-



$2T \times 2.5 + 100 \times 6 = T \times 4 + 200 \times 6$

$5T + 600 = 4T + 1200$

So, $T = 600N$

Therefore, the tension in the Cable at C is 600N.

It was assumed that the steel cables were light and inextensible.

(b) Resolving forces vertically:-

$600 + 1200 = 100 + W + 200$

$1800 = 300 + W$

so, $W = 1500N$

Therefore the steel rod has a weight of 1500N.

Example :-

A 100-lb vertical force is applied to the end of a lever which is attached to a shaft at O.

• Determine :-

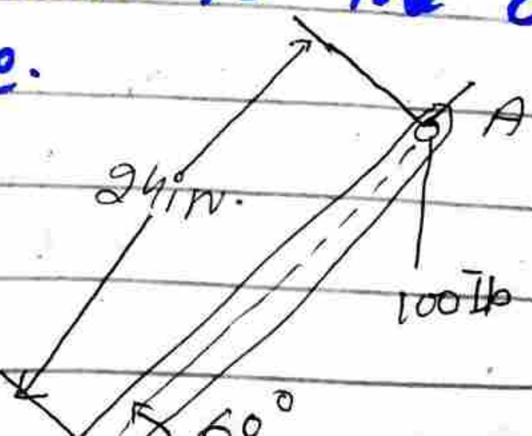
(a) Moment about O,

(b) Horizontal force at A which Creates the same Moment,

(c) Smallest force at A which produces the same Moment.

(d) Location for a 240-lb vertical force to produce the same moment,

(e) whether any of the forces from b, c, and d is equivalent to the orginal force.



Solution :-

(a) Moment about O is equal to the product of the force and the perpendicular distance between the line of action of the force and O. Since the force tends to rotate the lever Clockwise, the moment vector is into the plane of the paper.

$$M_o = Fd$$

$$d = (24\,in.)\cos 60^\circ = 12\,in.$$

$$M_o = (100\,lb)(12\,in).$$

$$M_o = 1200\,lb \cdot in$$

(b) Horizontal force at A that produces the same moment,

$$d = (24\,in)\sin 60^\circ = 20.8\,in.$$

$$M_o = Fd$$

$$1200\,lb \cdot in = f(28.8\,in)$$

$$f = \frac{1200\,lb \cdot in.}{20.8\,in.}$$

$$F = 57.7\,lb.$$

(c) The smallest force at A to produce the same moment occurs when the perpendicular distance is a maximum or when F is perpendicular to OA.

$$M_O = Fd$$

$$1200\ \text{lb} \cdot \text{in.} = F(24\ \text{in})$$

$$F = \frac{1200\ \text{lb} \cdot \text{in}}{24\ \text{in.}}$$

$$F = 50\ \text{lb.}$$

(d) To determine the point of application of a 240 lb force to produce the same moment,

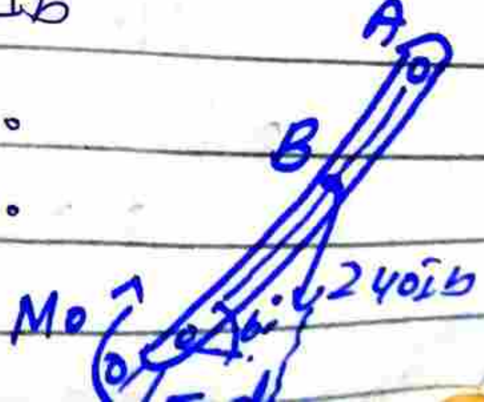
$$M_O = Fd$$

$$1200\ \text{lb} \cdot \text{in.} = (240\ \text{lb})d$$

$$d = \frac{1200\ \text{lb} \cdot \text{in}}{240\ \text{lb}} = 5\ \text{in.}$$

$$OB \cos 60^\circ = 5\ \text{in.}$$

$$OB = 10\ \text{in.}$$



(e) Although each of the forces in parts (b), (c), and (d) produces the same moment as the 100 lb force, none are of the same magnitude and sense, or on the same line of action. None of the forces is equivalent to the 100 lb force.

Lecture # 28
Couple and its Moments

Moment of a Couple:-

- Two forces F and $-F$ having the same magnitude, parallel lines of action, and opposite sense are said to form a couple.
- Moment of the couple,

$$\vec{M} = \vec{r}_A \times \vec{F} \times \vec{r}_B \times (-\vec{F})$$
$$= (\vec{r}_A - \vec{r}_B) \times \vec{F}$$
$$= \vec{r} \times \vec{F}$$

$$M = rF\sin\theta = Fd$$

(e) Although each of the forces in parts (b), (c), and (d) produces the same moment as the 100 lb force, none are of the same magnitude and sense, or on the same line of action. None of the forces is equivalent to the 100 lb force.

Lecture # 28
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$$= (\vec{r}_A - \vec{r}_B) \times \vec{F}$$

$$= \vec{r} \times \vec{F}$$

$$M = rF\sin\theta = Fd$$

The moment vector of the couple is independent of the choice of the origin of the coordinate axes, it is a free vector that can be applied at any point with the same effect.

Two couples will have equal moments if

- $F_1 d_1 = F_2 d_2$
- the two couples lie in parallel planes, and the two couples have the same sense or the tendency to cause rotation in the same direction.

- Consider two intersecting planes P_1 and P_2 with each containing a couple.

$\vec{M}_1 = \vec{r} \times \vec{F}_1$ in plane P_1
$\vec{M}_2 = \vec{r} \times \vec{F}_2$ in plane P_2

- Resultants of the vectors also form a Couple
$$\vec{M} = \vec{r} \times \vec{R} = \vec{r} \times (\vec{F}_1 \times \vec{F}_2)$$

- By varigon's theorem
$$\vec{M} = \vec{r} \times \vec{F}_1 + \vec{r} \times \vec{F}_2$$
$$= \vec{M}_1 + \vec{M}_2$$

- Sum of two Couples is also a Couple that is equal to the vector sum of the two Couples

- A Couple Can be represented by a vector with Magnitude and direction equal to the moment of the Couple.

* Couple vectors obey the law of addition of vectors.

* Couple vectors are free vectors, the point of application is not significant

* Couple vector may be resolved into component vectors.

Example:-

Determine the Resultant Couple Moment of three couples acting on the plate.

$F_1 = 200N$, $F_2 = 450N$, $F_3 = 300N$

$d_1 = 4m$, $d_2 = 3m$, $d_3 = 5m$

$\gamma_1 =$, $\gamma_2 = 450N$

C_1:- $F_1 = 200N$

$d_1 = 4m$

$M_1 = -F_1 d_1 = -200 \times 4 = -800Nm$

C_2 :- $F_2 = 450N$.
$d_2 = 3m$
$M_2 = F_2 d_2 = 450 \times 3m$
$= 1350Nm$

C_3 : $F_3 = 300N$
$d_3 = 5m$
$M_3 = -F_3 d_3 = 300N \times 5m$
$= 1500Nm$

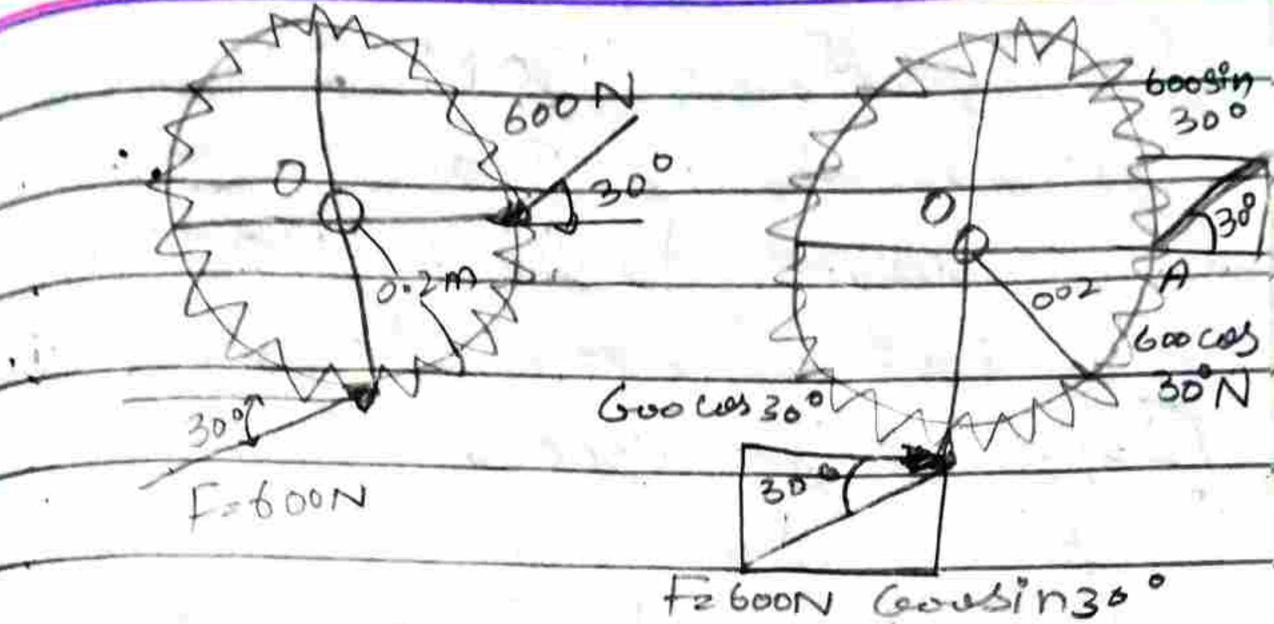
$M_R = -800 + 1350 - 1500$
$= -95\ Nm$

lecture # 29, 30

Applications of Couple and its Moments

Examples:-

(9) Determine the Magnitude and the direction of Couple moment acting on the gear as Shown in the fig.



Sol:- $F_y = F\sin\theta$

$= 600\sin 30°$

$F_x = F\cos\theta$

$= 600\cos 30°$

Moment about O;

$M_o = M_1 + M_2 = 600\cos 30° \times 0.2$

$- 600\sin 30° \times 0.2$

$= 103.9 - 60 = 43.9N$

Example:-

Given: A 2-D force System with geometry as shown.

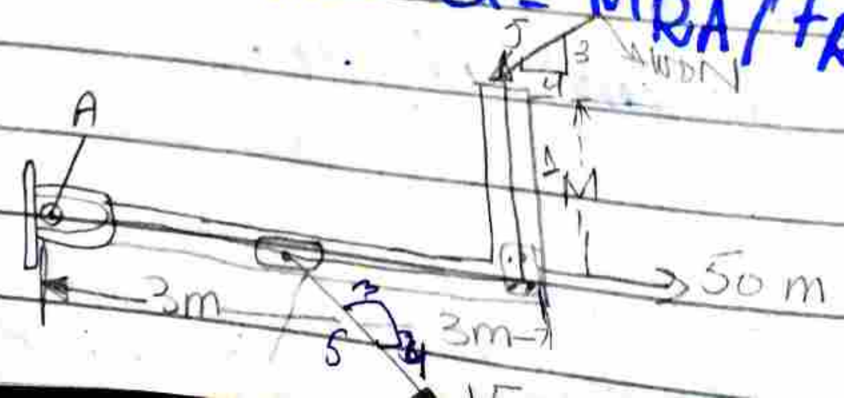
Find:- The Equivalent resultant force and Couple moment acting at A and then the equivalent Single force location measured from A.

Plan:

① Sum all the x and y Components of the forces to find F_{RA}.

②. Find and Sum all the Moments resulting from moving each force Component to A.

③ Shift F_{RA} to a distance d Such that $d = M_{RA}/F_{Ry}$



Solution:-

$$\rightarrow \sum F_{Rx} = 150(3/5) + 50 - 100(4/5)$$

$$= 60N$$

$$\downarrow \sum F_{Ry} = 150(4/5) + 100(3/5)$$

$$= 180N$$

$$\curvearrowleft M_{RA} = 100(4/5)1 - 100(3/5)6$$

$$- 150(4/5)3 = -640Nm$$

$$F_R = (60^2 + 180^2)^{1/2} = 190n$$

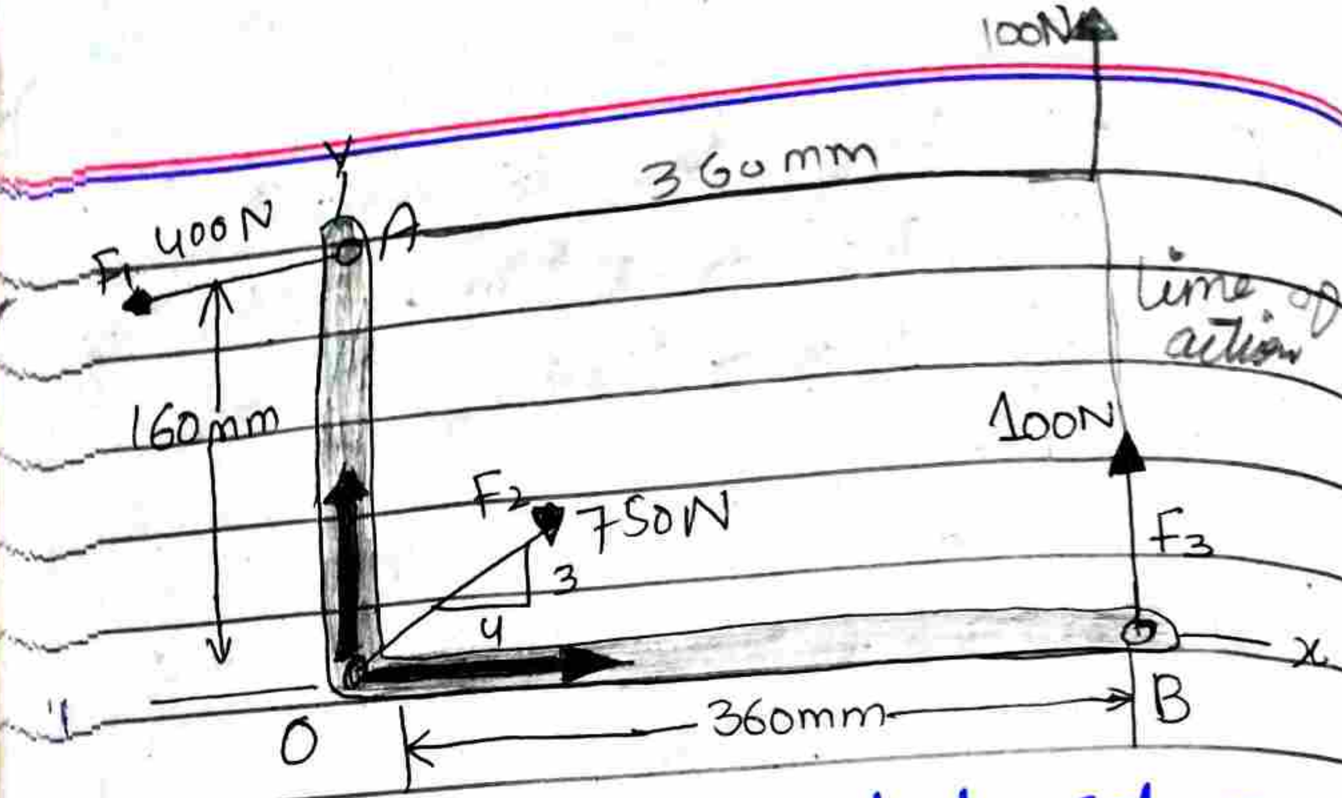
$$\theta = \tan^{-1}(180/60) = 71.6^\circ$$

The equivalent single force F_R can be located at a distance d measured from A.

$$d = M_{RA} / F_{Ry} = 640/180 = 3.56m$$

Example:-

Replace the three forces show with an equivalent force-couple system at A.



To find the equivalent set of forces at A.

$$\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.87^\circ$$

$$R_x = \Sigma F_x$$
$$= -400N + 750N\cos(36.87^\circ)$$
$$= 200N$$

$$R_y = \Sigma F_y = 750N\sin(36.87^\circ) + 100N$$
$$= 556$$

$$R = \sqrt{200^2 + 550^\circ} = 583N$$

$$\alpha = \tan^{-1}\left(\frac{550}{200}\right) = 70^\circ$$

To find the moments about point A.

using line of action of force

at B. The force can be moved along the line of action until it reaches perpendicular distance from A;

$$|\vec{M_1}| = |\vec{F_B}|\, d$$
$$= 100N\,(360mm)$$
$$= 36000\ N\text{-}mm.$$

using the line of action for each component, their moment contribution can be determined.

The force at O can be broken up into its two components in the x and y-direction.

$$F_x = 750N\cos(36.87^\circ)$$
$$= 600N$$
$$F_y = 750N\sin(36.87^\circ)$$
$$= 450N$$

using the line of action for $f(x)$ component d is 160 mm.

$$|\vec{M_2}| = |\vec{F_{ox}}| d$$
$$= 600N (160mm)$$
$$= 96000 \text{ N-mm}$$

F_y Component is 0 since in line with A.

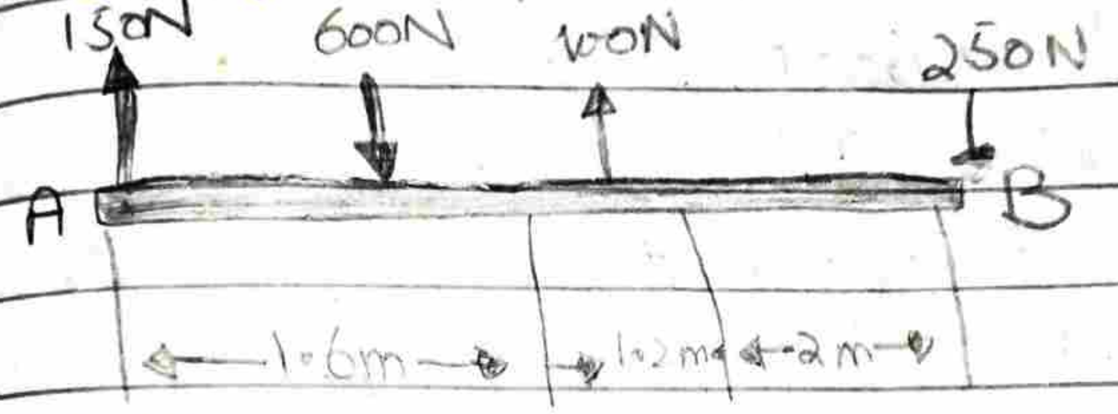
$$M_A = \sum M_i$$
$$= M_1 + M_2 + M_3$$
$$= 36000 \text{ N-mm} + 96000 \text{ N-mm} + 0 \text{ N-mm}$$
$$= 132000 \text{ N-mm}$$
$$= 132 \text{ N-m}.$$

Example :-

for the beam given in the fig.

(a) Compute the resultant force for the force shown and the resultant Couple for the moments of the forces about A.

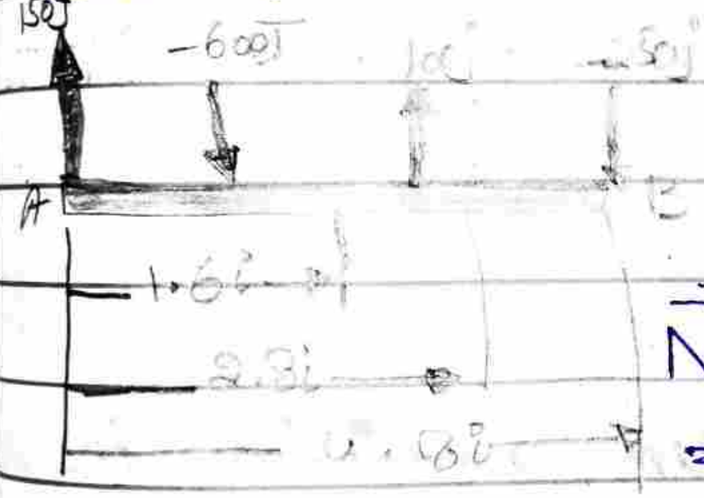
(b) find an equivalent force-Couple System at B based on the force-Couple System at A.



Solution :-

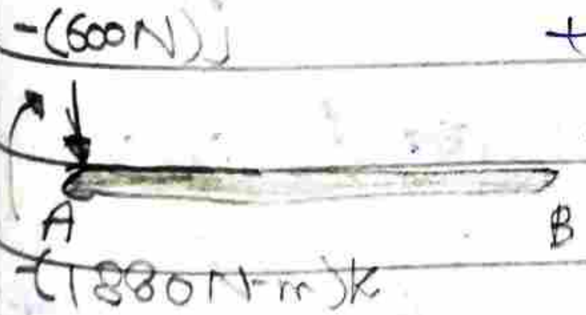
(a) $\vec{R} = \sum \vec{F} = (150\,N)\hat{j} - (600\,N)\hat{j} + (100\,N)\hat{j} - (250\,N)\hat{j}$

$$\boxed{\vec{R} = -(600\,N)\hat{j}}$$



$$\vec{M}^R_A = \sum(\vec{r} \times \vec{F})$$

$$= (1.6\hat{i}) \times (-600\hat{j}) + (2.8\hat{i}) \times (100\hat{j}) + (4.8\hat{i}) \times (-250\hat{j})$$



$$\boxed{\vec{M}^R_A = -(1880\,N\text{-}m)\hat{k}}$$

(b) The force is unchanged by the movement of the force-Couple System from A to B:

$$\vec{R} = -(600N)\hat{j}$$

The Couple at B is equal to the moment about B of the force-Couple ~~force~~ System found at A.

$$\vec{M}_B^R = \vec{M}_A^R + \vec{r}_{B/A} \times \vec{R}$$

$$= -(1880\,N\text{-}m)\,\vec{k} + (-4.8m)\,\hat{i} \times (-600N)\,\hat{j}$$

$$= -(1880\,N\cdot m)\,\hat{k} + (2880\,N\cdot m)\,\hat{k}$$

$$\vec{M}_B^R = +(1000\,N\text{-}m)\,\hat{k}$$

Lecture #31

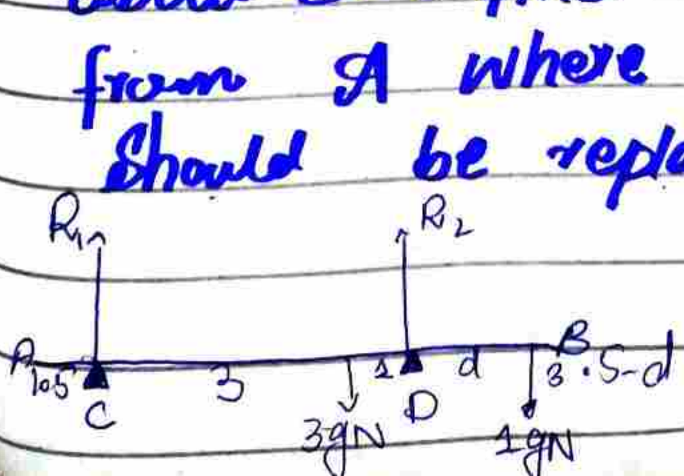
Tilting of a particle. (d)

Tilting:-

if a rod is resting on two supports at A & B and is on the point of tilting about A, then the reaction force at B is 0.

Question:-

A 9m uniform rod AB is resting on two Supports at C and D, where C is 1.5m from A & D is 3.5m from B. The Mass of the rod is on the point of tilting about D. Find the distance from A where this Mess should be replaced.



when the rod is about to tilt:-

$R_1 = 0$

Taking moments about D:

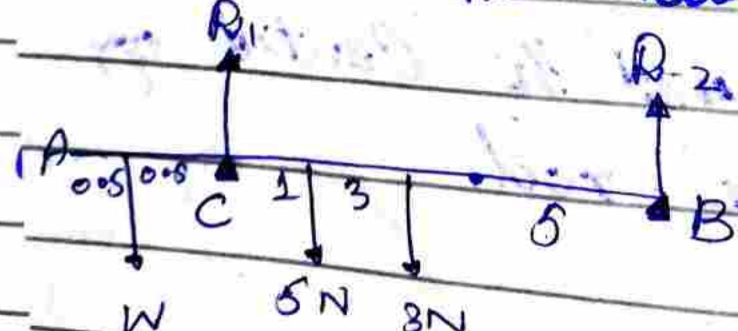
$3g \times 1 = 1g \times d$

$3 = 1d$

Therefore the 1kg mass should be placed 8.5m from A.

Question:-

A uniform rod AB of length 10m and weight 3N rests on two supports at B & C, where C is 1m from A. A weight of 5N is attahed 0.5m from A. the rod is on the point of tilting about C. Find the weight of this load in Newtons.



Taking Moments about C:

$$0.5W = 5 \times 1 + 3 \times 4$$

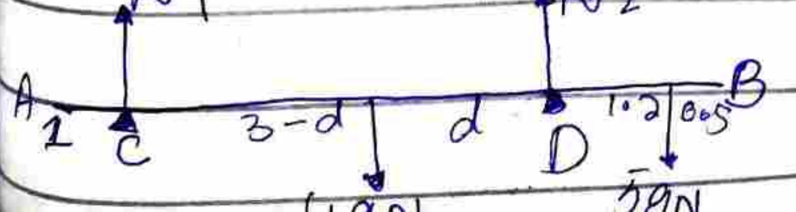
$$0.5W = 17$$

$$W = 34N$$

Therefore the load must have a weight of 34N

Question :-

A non-uniform rod AB of length 6m and Mass 4Kg is resting on two Supports C & D, where C is 1m from A & D is 2m from B. When a Mass of 5Kg is attached at a point 0.8m from B, the rod is on the point of tilting about D. Calculates the distance between A and the Centre of mass of the rod.



Taking Moments about D:-

$$4g \times d = 5g \times 1.2$$

$$4g = 6$$

$$d = 1.5m$$

Therefore the Centre of mass of the Rod is 2.5m from A.

Example:-

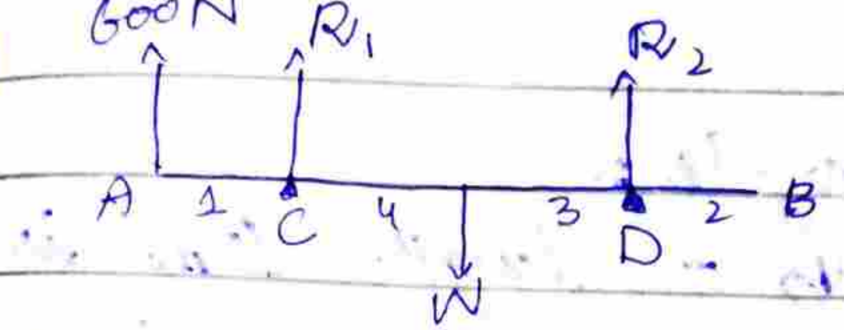
A log AB of length 10m is placed on two smooth supports at C & D where C is m from A and D is 2m from B. wh a force of 600N is applied to the log vertically upwards at A, the log is on the point of tilting about D.

Initially, the log is modelled as a uniform rod.

(a) Calculate an estimate of the weight of the log.

The force at A is now removed and a force of 750 N is applied to the log vertically upwards at B. The log is now on the point of tilting around about C.

ⓐ



Taking Moments about D :-

$600 \times 8 \times 3W$

$3W = 4800$

$W = 1600\,N$

$\boxed{\text{Remember } R_1 = 0}$

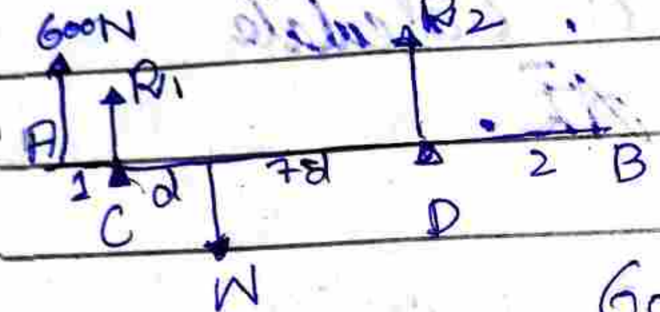
Therefore an estimate of the weight of log is 1600N

ⓑ Remodelling the log as non-uniform, calculate a revised estimate of the weight of the log.

Taking Moments about C:-

$$W \times d = 750 \times 9$$

$$Wd = 6750 \rightarrow (i)$$



Taking Moments about D:-

$$600 \times 8 = W \times (7-d)$$

$$7W - Wd = 4800 \rightarrow (ii)$$

$$7W - Wd = 4800 \quad \Rightarrow \text{By } (i) + (ii)$$

$$+ \quad Wd = 6750$$

$$7W = 11550$$

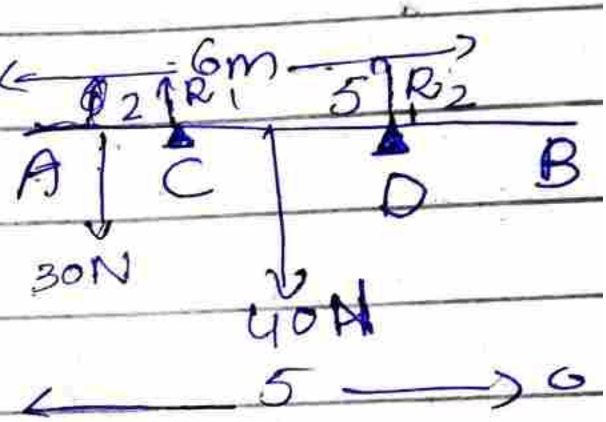
$$\text{So, } W = 1650$$

Therefore, a revised estimate of the weight of the log is **1650 N**.

Example :-

A uniform bar AB of length 6m and weight 610N is resting in a horizontal position on supports at points C and D where AC = 2m and AD = 5m. When a particle of weight 30N is attached to the bar at point $\bar{E}$ the bar is on the point of tilting about C. Calculate the distance $A\bar{E}$.

$R_2 = 0$ (on the point of tilting)



Moments about C

$30(2-d) = 40(1)$

$60 - 30d = 40$

$30d = 20$

$d = \frac{2}{3}$

Lecture # 32

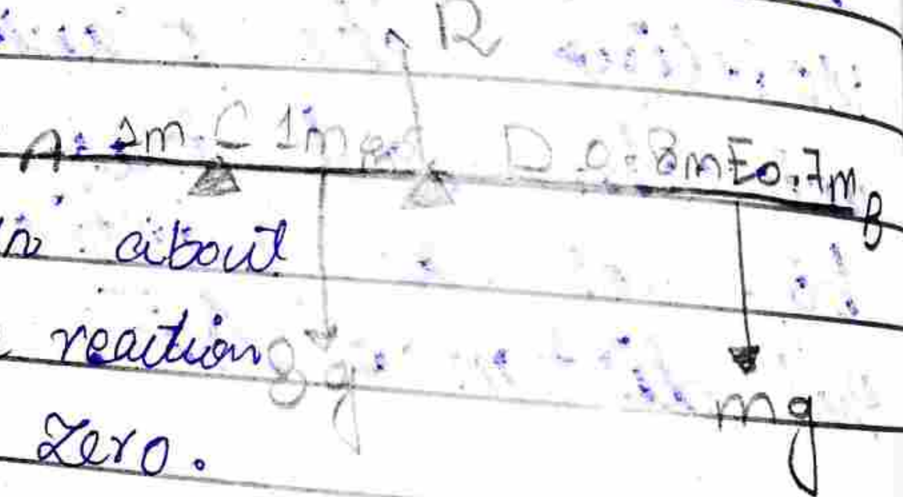
Applications of Tilting of a particle.

Example:-

A uniform rod AB has length 4 m and Mass 8kg. it is resting in a horizontal position on supports at points C & D were $AC = 1m$ & $AD = 2.5m$. A particle of mass m kg is placed at point E where $AE = 3.3$ m. Given that rod is about to tilt about D, Calculate the value of m.

Solution:-

if a rod is about to turn about D then the reaction at C is zero.



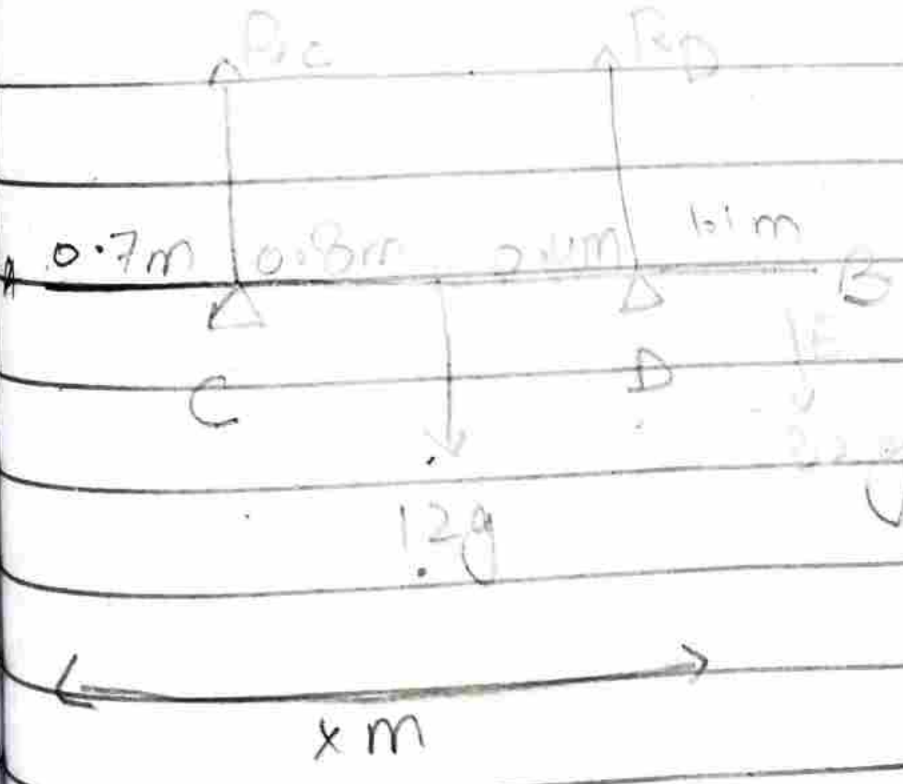
Taking moments about points D:

$$8g \times 0.5 = mg \times 0.8$$

$$\Rightarrow m = 5$$

Example :-

A plank AB of Mass 12 kg and length 3m is in equilibrium in a horizontal position resting on Supports at C and D where AC = 0.7 m and DB = 1.1 m. A boy of Mass 32 kg stands on the plank at point $\bar{x}$. The plank is about to tilt about D. By Modeling the plank as a uniform rod ~~has~~ and the boy as a particle, Calculate the distance $A\bar{x}$.



Let the distance $A\bar{x}$ be x m.
If the plank is about to tilt about D then $R_c = 0$.
Taking moments about D:

$$12g \times 0.4 = 32g \times (x - 1.9)$$

$$12 \times 0.4 = 32x - 32 \times 1.9$$

$$32x = 4.8 + 60.8 = 65.6$$

$$\Rightarrow x = 65.6 \div 32 = 2.05m$$

$\bar{x}$ is 2.05m from A.

Example:-

A Non-uniform rod AB of length 5m and Mass 15Kg rests horizontally suspended from the Celing by two vertical strings attached to C & D, where AC = 1m and AD = 3.5m

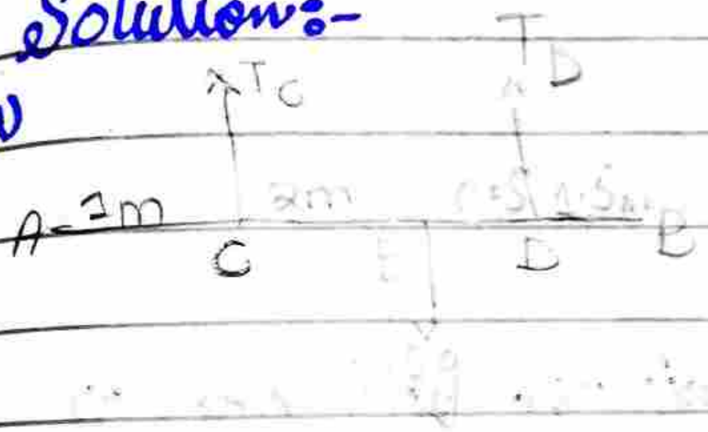
(a) Given that the Centre of Mass is at $\bar{x}$ where $A\bar{x}$ = 3m, Find the Magnitude of the tensions in the strings.

When a particle of Mass 10kg is attached to the rod at F the rod is just about to rotate about D.

(b) Find the distance AF.

Solution:-

(a)



Taking Moments about C:

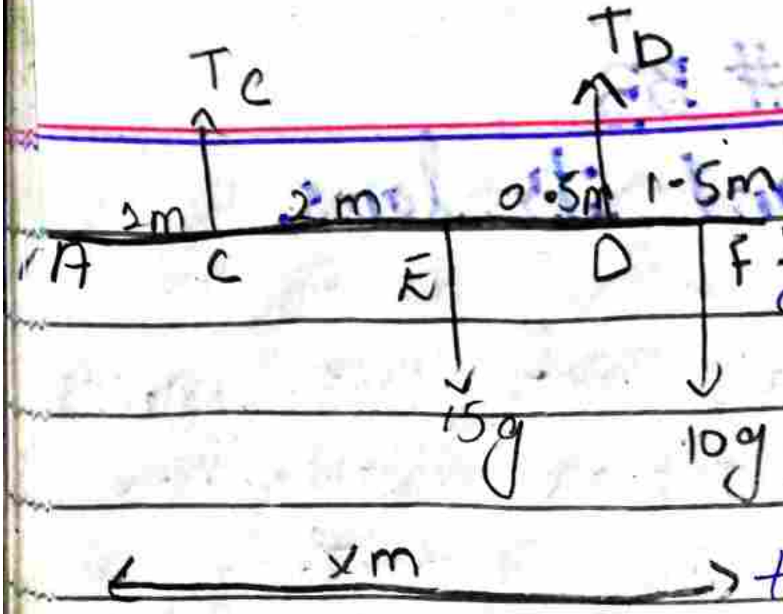
$T_D \times 2.5 = 15g \times 2$

$2.5T_D = 30g$

$\Rightarrow T_D = 12g = 118N$ (3s.f.)

R(↑)

$T_C + T_D = 15g$, $T_C = 3g = 29.4N$



when the particle attahed at F, T_C is zero because the rod is about to rotate about D.

let the distance AF = xm.

Taking Moments about D:

$$15g \times 0.5 = 10g \times (x - (1+2+0.5))$$
$$= 10g \times (x - 3.5)$$
$$7.5g = 10gx - 35g$$
$$42.5 = 10x$$
$$x = 4.25$$

The distance AF is 4.25m.

Lecture # 33
Friction and its laws.

* Friction is a force that opposes Motion and occurs when two Surface are in Contact.
* it will therefore act in the opposite direction to Motion.

What is friction

* friction is a force.
* A frictional force can exist when two substances Contact each other.
* The Molecules of each Surface interact according to Newton's law of Motion.
* Friction always opposes Motion, it is opposite to the direction of velocity.
* if there is no Motion, then friction opposes the Sum of all the other forces

which are parallel to the surface in contact.

Types of Friction

① **Dry Friction:-**
occurs between the non-lubricated surfaces of solid objects.

② **Fluid friction:-**
occurs with fluids, or lubricated surfaces.

③ **Static Frictions:-** Types of Dry Friction
When dry friction acts between two surfaces that are not moving relative to each other.

④ **Dynamic friction:-** Type of Dry Friction
When dry friction acts between two surfaces that are moving relative to each other.

Laws of Friction

* When two surfaces are in Contact, the force of Friction opposes the Motion.
* if two Surfaces are in equilibrium, the force of friction is just sufficient to prevent Motion and can be found by resolving the forces parallel to the surface.
* Friction force will not be larger than necessary to prevent Motion.
* The size of friction force is limited. once the limited friction is reached and the acting force is great enough to overcome friction then motion will occur. At such point equilibrium is broken. That is when:-

Limiting Friction $f = \mu R$

Contact force

* force that occurs between objects that are in Contact with each other.
* Contact forces Can be resolved into Components that are perpendicular and parallel to the Surfaces in Contact.
* The perpendicular Component is Called the **Normal force.**
* The parallel, Component is called friction.

Friction and the Normal Force.

:

* The Maximum frictional force is proportional to the normal Contact force.
* An increase in the Normal force results in an increase in the Maximum friction.
* This is because the Molecules on the two surfaces are pushed together more thus increasing their interactions.

The Maximum value of friction is Calculated by using the formula:-

$$f_{max} = \mu R$$

μ → Cofficient of friction.

R → Reaction

dependent upon the Material of the Surface and object.

The Friction acting on the particle is f_{max} when:

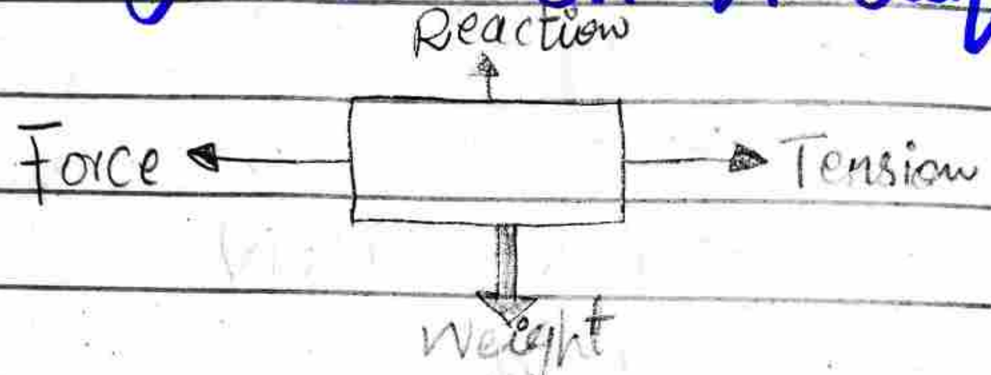
1. The particle is in "limiting equilibrium"
2. The particle is Moving.

The following equation is used for friction when an object is at rest and is not Moving:-

$$f \leq \mu R.$$

* The Magnitude of frictional force can change to ensure equilibrium.
* As the force pulling the sleigh increases, so will the friction to ensure equilibrium.

Friction On A Surface



- Resolving vertically, $R = mg$, where $g = 9.8\ m/s^2$
- Resolving Horizontally, $f = T$ for equilibrium at rest.
- if $T > f$ there will be a resultant force, which will produce an acceleration in the positive direction.

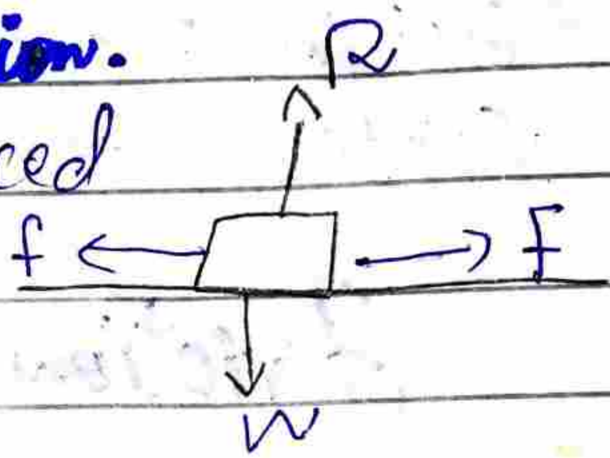
$\Rightarrow$ force $= ma = T - F$ according to

Newton's Second law of Motion

where $f = \mu R$.

Limiting equilibrium & Maximum Friction.

A body is placed on a rough surface:-



$$\text{Mass} = 10\,kg$$

$$W = mg$$

$$= 10 \times 9.8 = 98N$$

$$R = W = 98N$$

$$\mu = 0.5$$

$$f_{max} = \mu R$$

$$= 0.5 \times 98$$

$$= 49N$$

Mth.304

Lecture # 34

Applications of frictions.

Angle of friction:-

Angle of limiting friction is defined as the angle between the resultant reaction (of limiting friction and normal reaction) and the normal to the plane on which the Motion of the body is impending.

Angle of Friction:-

$$F \propto N$$

$$F_{Max} = \mu N$$

$$\mu = \frac{F_{Max}}{N}$$

$\tan \phi = (F_{max})/N = \mu$

Angle ϕ is Known as Angle of limiting Friction

Friction Examples:-

Example # 01:-

A horizontal rope is attahed to a crate of Mass 70kg at rest on of flat surface. The Coefficient of friction between the floor and the crate is 0.6. Find the Maximum force that the rope can exert on the crate without Moving it.

Given:-

$m = 70kg$ and $\mu = 0.6$

$F < \mu R$ and $R = mg$

Find:-

$F = ?$ $R = ?$

$R = mg$

$R = 70 \times 9.8 = 686N$

$F < \mu R$

$F \leq 0.6 \times 686$

$F < 411.6\ N$

$F_{max} = 411.6$

Example:-

A block of Mass 15kg rests on a rough horizontal Surface. Coefficient of friction between block and the plane is 0.25. Calculate the Friction force acting on the block when a horizontal force of 50N act on the block. State whether the block will move.

Given:- $m = 15kg$ and $\mu = 0.25$

with horizontal force $= 50N$

using :- $R = mg$

$$R = 15 \times 9.8$$

$$= 147N.$$

using:- $F = \mu R$

$$= 0.25 \times 147$$

$$= 36.75N$$

Sin $F < 50N$ so the block moves.

Example :-

A Horizontal rope is attached to a crate of Mass 50kg at rest on a flat surface. The coefficient of friction between the floor and the crate is 0.4. Tension in the rope is 100 N. Find the frictional force exerted on the crate.

Givens:-

$m = 50$ kg and $\mu = 0.4$

$R = mg$

Find :-

$f = ?$ $R = ?$

Let T be the Tension in the rope

$R = mg$

$R = 50 \times 9.8 = 490N$

$\mu R = 0.4 \times 490 = 196$

Frictional force $f = T = 100N$

Example :-

A block of Mass 400kg rests in limiting equilibrium on horizontal ground. A force of Magnitude 2000N acts at angle of 15 degree to the upward vertical. Find the Coefficient of friction between the block and the ground, Correct to 2 Significant finguores.

Sol:-

$$m = 400kg$$

$$W = mg$$

$$W = 400 \times 10$$

$$= 4000N$$

$$R + 2000\sin 75 = 4000$$

$$R = 4000 - 2000\sin 75^\circ$$

$$= 2068.15N$$

$$\mu R = 2000\cos 75^\circ$$

$$\mu(2008.15) = 2000\cos 75^\circ$$

$$\mu = 0.25.$$

lecture # 35

Equilibrium of a particle on a rough horizontal plane.

Examples:-

A particle of mass 2kg is resting on a rough horizontal Surface. The coefficient of friction between the particle and the surface is 0.1. A light inextensible string is attached to the right of the particle at an angle of 30° to the horizontal if the particle is on the point of moving to the right, find the tension in the string.

Sol:-

Resolving vertically:

$$R + T\cos 60^\circ = 2g$$

Resolving horizontally:

$$F = T\cos 30^\circ$$

$F = \mu R$

$\therefore T\cos 30^\circ = 0.1 \times (2g - T\cos 60^\circ)$

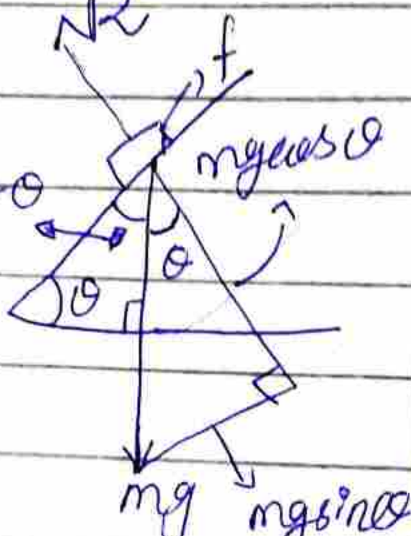
$\therefore T(\cos 30^\circ + 0.1\cos 60^\circ) = 0.2g$

$T = 2.14$ (to 3 s.f.)

Lecture # 36.

Equilibrium of a particle on a rough inclined plane.

Conditions of Equilibrium for a particle on rough Inclined plane.



Mass = m kg

$W = mgn$

μ = co-efficient of friction.

$f_{max} = \mu R$

$f_{max} = \mu(mg\cos\theta) = \mu mg\cos\theta$.

$f = mg\sin\theta$ (equilibrium condition)

for limiting eq:-

$f_{max} = mg\sin\theta$

$\mu mg\cos\theta = mg\sin\theta$

$\mu = \frac{\sin\theta}{\cos\theta} = \tan\theta$

Example :-

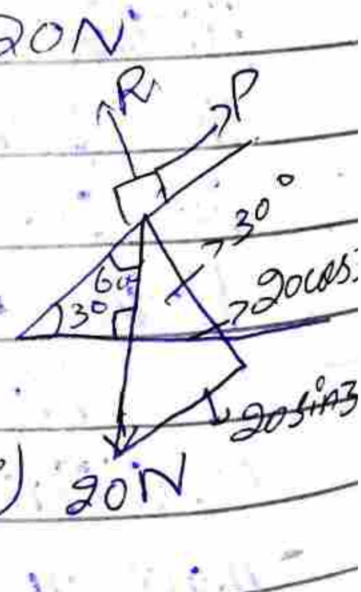
A box of Mass 2kg is placed on a plane which is inclined at an angle of 30 degrees with the horizontal. The plane is rough and the coefficient of friction between the box and the plane is 0.5. The box is kept in equilibrium by applying a horizontal force P Newton to it. Find the least and Maximum value of the force P which enable the box to remain in equilibrium.

Sol:- mass = 2kg

$W = mg = 2(10) = 20N$

$\mu = 0.5$

$R = 20\cos 30°$



$P + \mu R = 20\sin 30°$

$P + 0.5(20\cos 30°) = 20\sin 30°$

$P = 20\sin 30° - 0.5(20\cos 30°)$

$P = 1.34\ N$

Max force:-

$$P = \mu R + 20\sin 30^\circ$$

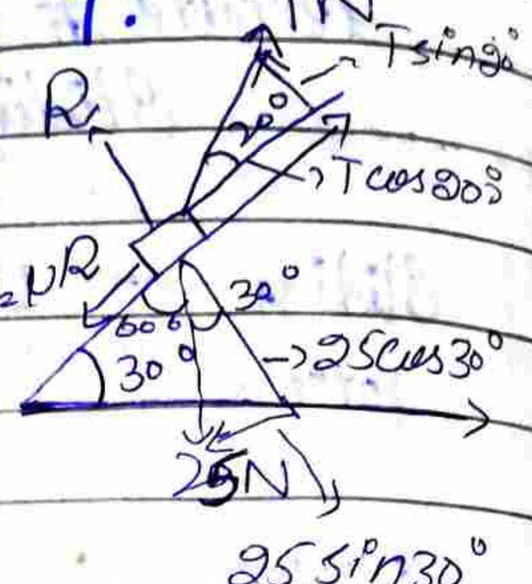
$$= (0.5)(20\cos 30^\circ) + 20\sin 30^\circ$$

$$P = 18.7\ N$$

Example:-

A block of mass 2.5Kg is placed on a plane which is inclined at angle of 30 degrees to the Horizontal. The block is Kept in equilibrium by a light String making an angle of 20 degree above a line of greatest slope. The Tension in the string is TN, . The cofficient of Friction between the block and the plane is 1/4. The block is in limiting between the block and the Plane is 1/4. The block is in limiting equilibrium and is about to move up the plane.

Find the value of T.



Sol:-

mass = m = 2.5 kg

$W = mg = 2.5 \times 10$

$= 25N$

$\mu = \frac{1}{4} = 0.25$

Parallel to the plane:-

$T\cos 20^\circ = 25\sin 30^\circ + \mu R$

$= 25\sin 30^\circ + \frac{1}{4}R \longrightarrow (1)$

Perpendicular to the plane:-

$R + T\sin 20^\circ = 25\cos 30^\circ$

$R = 25\cos 30^\circ - T\sin 20^\circ \longrightarrow (2)$

Put (2) in (1)

$T\cos 20^\circ = 25\sin 30^\circ + \frac{1}{4}(25\cos 30^\circ - T\sin 20)$

$T\cos 20^\circ + \frac{1}{4}T\sin 20^\circ = 25\sin 30^\circ + \frac{25}{4}\cos 30^\circ$

$T(1.025) = 17.913$

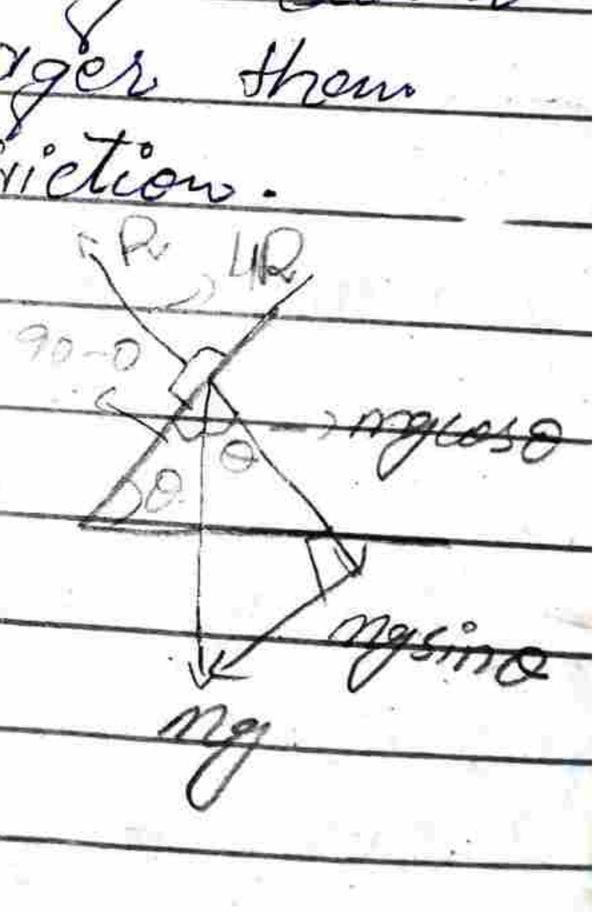
$T = \frac{17.913}{1.025} = \boxed{17.5N}$

Lecture # 37
Sliding of particle.

Sliding of particle without external force:-

Exam

An object will slide down a plane if the component of its weight acting down the plane is larger than the Maximum friction.



$mg \sin\theta > \mu R$

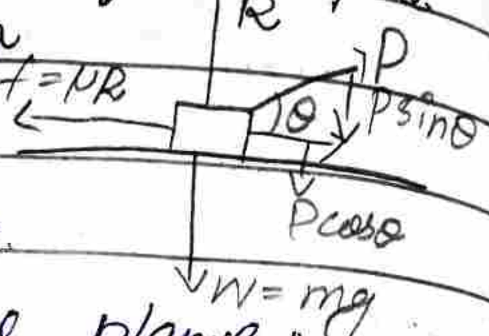
$mg \sin\theta > \mu (mg \cos\theta)$

$\sin\theta > \mu \cos\theta$

$\boxed{\tan\theta > \mu}$

Least force to slide a particle on Horizontal plane..

let P be the magnitude of the force to drag a particle on a rough horizontal plane and let it make an angle theta with the plane. Other fores are.



weight of the particle acting downward.

Normal reaction R of the plane acting vertically upward friction of the plane acting along the plane.

$$W = mg$$

$$\rightarrow \quad P\cos\theta = \mu R \longrightarrow (1)$$

$$\uparrow \quad R + P\sin\theta = mg$$

$$R = mg - P\sin\theta \longrightarrow (2)$$

$$P\cos\theta = \mu(mg - P\sin\theta)$$

$$P\cos\theta = \tan\lambda\,(mg - P\sin\theta)$$

$$P\cos\theta = \frac{\sin\lambda}{\cos\lambda}(mg - P\sin\theta)$$

$P\cos\theta\cos\lambda = \sin\lambda\, mg - \sin\lambda\, p\sin\theta$

$P(\cos\theta\cos\lambda + \sin\lambda\sin\theta) = mg\sin\lambda$

$P(\cos(\theta-\lambda)) = mg\sin\lambda$

$$P = \frac{mg\,\cancel{\cos(\theta-\lambda)}\,\sin\lambda}{\cos(\theta-\lambda)}$$

$\max \cos(\theta-\lambda) = 1$

$\theta - \lambda = 0$

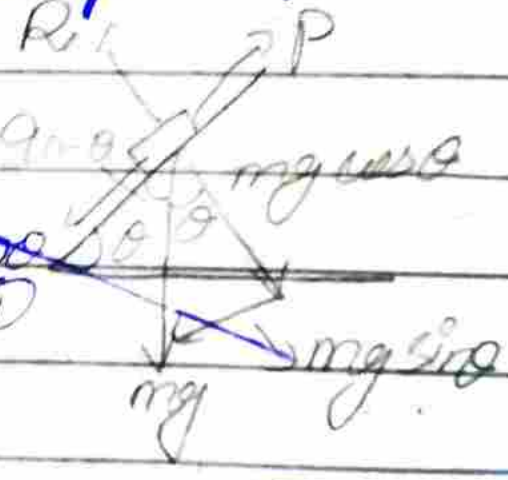
$\theta = \lambda$

$\boxed{P \text{ least} = mg\sin\lambda}$

least force to slide a particle up a rough inclined plane;

μ

mass $= m$



$P\cos\theta = \mu R + mg\sin\theta \rightarrow (1)$

$R + p\sin\theta = mg\cos\theta$

$R = mg\cos\theta - p\sin\theta \rightarrow (2)$

$P\cos\theta = \mu(mg\cos\theta - p\sin\theta) + mg\sin\theta$

$P\cos\theta = \mu mg\cos\theta - \mu p\sin\theta + mg\sin\theta$

$P\cos\theta = \frac{\sin\lambda}{\cos\lambda} mg\cos\theta - \frac{\sin\lambda}{\cos\lambda} p\sin\theta + mg\sin\theta$

$$P\cos\theta\cos\lambda = \sin\lambda\, mg\cos\theta - P\sin\lambda\sin\theta + mg\sin\theta\cos\lambda$$

$$P(\cos(\theta-\lambda)) = mg(\sin\theta\cos\lambda + \cos\theta\sin\lambda)$$

$$P = \frac{mg\sin(\theta+\lambda)}{\cos(\theta-\lambda)}$$

$\because$ P least $\cos(\theta-\lambda) = 1$

$$\theta - \lambda = 0$$

$$\theta = \lambda$$

P least $= mg\sin(\theta+\lambda)$

Lecture # 38

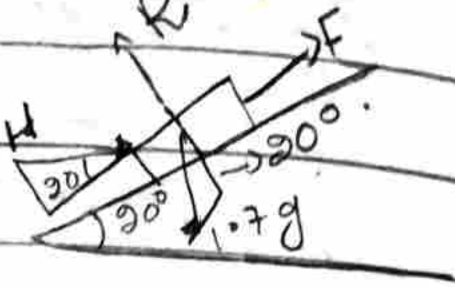
Applications of sliding of a particle.

Sliding of particle Examp #01:-

A particle of mass 1.7kg rests on a rough plane inclined at an angle of 20° to the horizontal. A Horizontal force, H, is applied to the particle. The coefficient of friction between the particle and the plane is 0.15. Find H if:-

(a) The particle is on the point of sliding down the plane.

(b) The particle is on the point of sliding up the plane.



(a) Resolving perpendicular to the plane:

$R = 1.7g\cos 20° + H\cos 70°$

Resolving parallel to the plane:

$F + H\cos 20° = 1.7g\cos 70°$

$\therefore F = 1.7g\cos 70° - H\cos 20°$

$F = \mu R$

$1.7g\cos 70° - H\cos 20° = 0.15(1.7g\cos 20° + H\cos 70°)$

$0.15H\cos 70° + H\cos 20° = 1.7g\cos 70° - 0.15 \times 1.7g\cos 20°$

$H(0.15\cos 70° + \cos 20°) = 1.7\cos 70° - 0.15 \times 1.7g\cos 20°$

$H = \dfrac{1.7g\cos 70° - 0.15 \times 1.7g\cos 20°}{0.15\cos 70° + \cos 20°}$

$H = 3.38$ (to 3 s.f.)

Therefore the Horizontal force is 3.38 N.

(b) Resolving perpendicular to the plane;

$R = 1.7g\cos 20° + H\cos 70°$

Resolving parallel to the plane;

$F + 1.7g\cos 70° = H\cos 20°$

$F = H\cos 20° - 1.7g\cos 70°$

$F = \mu R$

$H\cos 20° - 1.7g\cos 70° = 0.15(1.7g\cos 20° + H\cos 70°)$.

$H\cos 20° - 0.15H\cos 70° = 0.15 \times 1.7g\cos 20° + 1.7\cos 70°$

$H(\cos 20° - 0.15\cos 70°) = 0.15 \times 1.7g\cos 20° + 1.7g\cos 70°$

$$H = \frac{0.15 \times 1.7g\cos 20° + 1.7g\cos 70°}{\cos 20° - 0.15\cos 70°}$$

$= 9.06$ (to 3 s.f.)

Therefore the horizontal force is 9.06N.

Example #02:-

A particle of Mass 10kg is Held in equilibrium on a smooth plane inclined at an angle of 30° to the Horizontal by means of a light inextensible string acting at an angle of 10° to the plane.

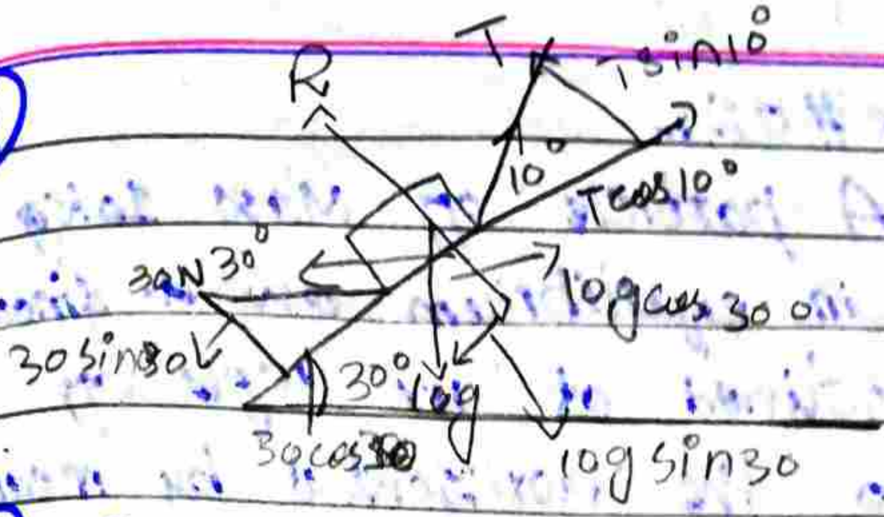
(a) Draw a diagram showing all the forces acting on the particle.

(b) Calculate the tension in the string.

A Horizont force of 30N is applied to the particle to try to make it slide up the slope. The tension in the string is adjusted so that the particle remains in equilibrium.

(c) Calculate the new tension in the string.

(a)



(b) Resolving parallel to the plane:

$T \cos 10° = 10g \cos 60°$

$T = 10g \cos 60° / \cos 10°$

$= 49.8$ (to 3sf)

Therefore the tension in the String is 49.8N.

(c) Resolving parallel to the plane:

$T \cos 10° + 30 \cos 30° = 10g \cos 60°$

$T = 10g \cos 60° - 30 \cos 30° / \cos 10°$

$T = 23.4$ (to 3SF).

Therefore the tension in the String is 23.4N.

Example #03:-

A particle of Mass m kg lies on a rough plane inclined at an angle of θ to the Horizontal. The particle is Held in equilibrium by Means of a light inextensible String held at an angle of 30° to the plane.

The coefficient of Friction between the plane and the particle is 0.25 and the particle is about to Slide up the plane.

Show that the tension in the String is $2mg\left(\frac{\cos\theta + 4\sin\theta}{4\sqrt{3}+1}\right)$

Resolving perpendicular to the plane,

$$R + T\cos 60 = mg\cos\theta$$

$$R = mg\cos\theta - \tfrac{1}{2}T$$

using $F = \mu R$

$F = \frac{1}{4} mg\cos\theta - \frac{1}{8}T$

Resolving parallel to the plane,

$F + mg\cos(90-\theta) = T\cos 30$

$\frac{1}{4} mg\cos\theta - T + mg\sin\theta = T\cos 30$

$= \frac{1}{4} mg\cos\theta \div T + mg\sin\theta$

$= \frac{\sqrt{3}}{2} T + \frac{1}{8} T$

$= \left(\frac{4\sqrt{3}+1}{8}\right) T$

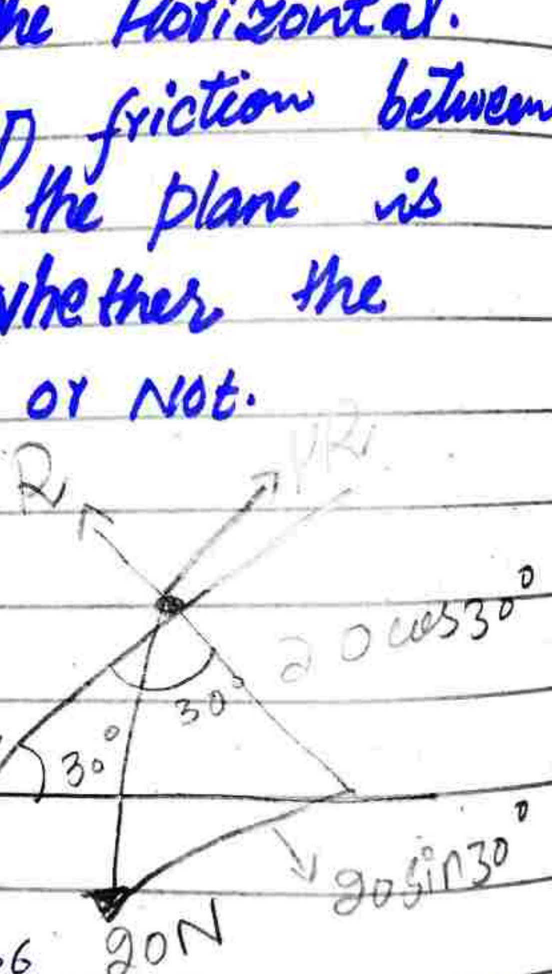
$T = 2mg\left(\frac{\cos\theta + 4\sin\theta}{4\sqrt{3}+1}\right)$ are required

Lecture # 39

Examples of sliding and friction.

Example: A particle of Mass 2kg is placed on a rough inclined plane which is at an angle of 30 derees withe Horizontal. The coefficient of friction between the particle and the plane is 0.5. Determine whether the particle will slide or Not.

Sol:-



$20\sin 30^\circ > \mu R$

$20\sin 30 > 0.5 \times 20\cos 30^\circ$

$20\frac{1}{2} > 10 \times \sqrt{3}/2$

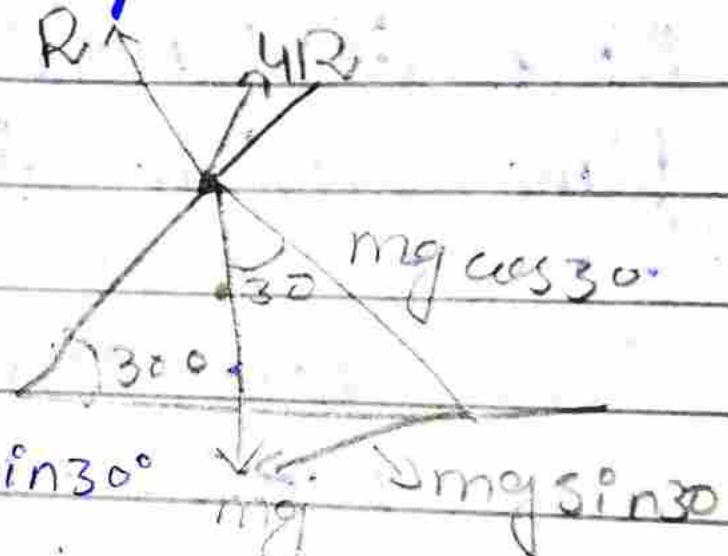
$10 \geq 10 \times 0.866$

$10 > 8.66$ which is true.

particle will slide.

Example:-

A particle is placed on a rough inclined plane which is angle of 30° with the Horizontal. Find the set of values of coefficient of Friction for the particle to slide.



for Sliding;

$\mu R < mg \sin 30^\circ$

$\mu / mg \cos 30^\circ < mg \sin 30^\circ$

$\mu < \frac{\sin 30^\circ}{\cos 30^\circ}$

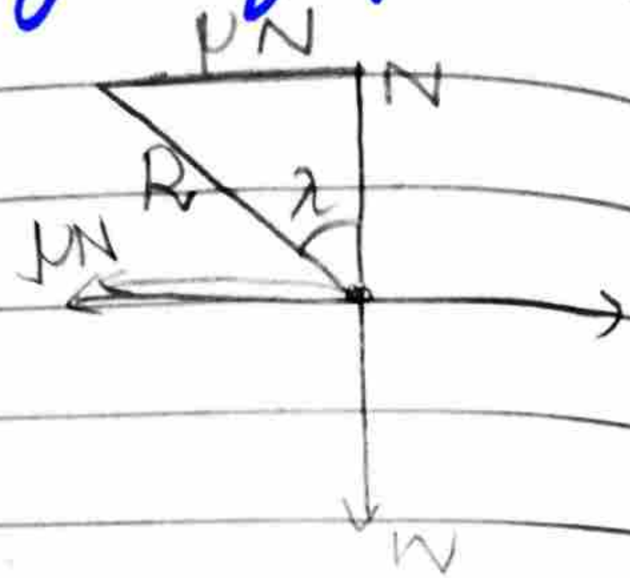
$\mu < \tan 30^\circ$

$\mu = 0.577.$

Example:-

Angle of friction.

A particle of mass 2kg is placed on a rough horizontal plane whose coefficient of friction is 0.4. Find the angle of friction.



$\tan\lambda = \frac{\mu N}{N}$

$\mu = \tan\lambda$

$\tan\lambda = \mu$

$\Rightarrow \lambda = \tan^{-1}\mu$

$\lambda = \tan^{-1}(0.4)$

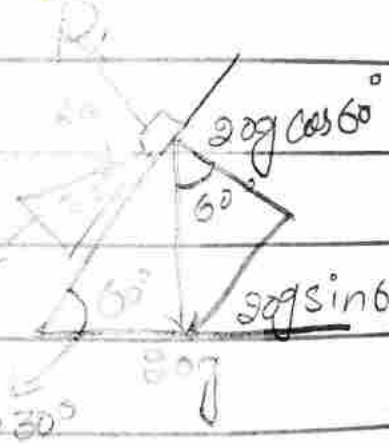
$\lambda = 21.8^\circ$

for More examples watch video lecture on LMS.

Lecture # 43

Applications of Workdone

Example:-
A particle of Mass 20kg is Supported on a Smooth plane is inclined at 60 degress to the horizontal by a force of Magnitude X Newton which makes an angle of 30 degrees with the plane. Find x and also the reaction of the plane on the particle.



principle of virtual work;

$x \cos 30^\circ - \partial s - 20g \sin 60^\circ \delta s = 0$

$x \cos 30 = 20g \sin 60^\circ$

$x = 20g$

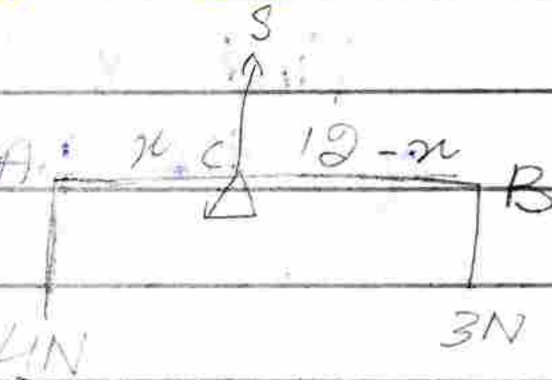
$R \cdot \delta y - x \sin 30^\circ \partial y - 20g \cos 60^\circ \delta y$

$R = x \sin 30^\circ + 20g \cos 60^\circ$

$= 20g \times \frac{1}{2} + 20g \times \frac{1}{2} = 20g.$

Example:-

A light thin rod, 12m long, Can turn in a vertical plane about one of its points which is attached to a pivot. if wights of 3N and 4N are suspended form its ends, it rests in a horizontal position. Find the position of the pivot and its reaction on the rod.



equation of V.W

$\delta \Omega$.

$4x \cdot \delta\Omega - 3(12-x)\,\delta\Omega = 0$

$4x - 36 + 3x = 0$

$7x - 36 = 0 \Rightarrow x = \frac{36}{7} = 5\frac{1}{7}$

$AC = 5\frac{1}{7}$ m, $BC = 12 - \frac{36}{7}$

$= \frac{48}{7} = 6\frac{6}{7}$ m

$\delta y \Rightarrow 5\partial y - 4\delta y - 35y = 0$

$S - 7 = 0$

$S = 7$