

The group G is a copy of the group H , if the function

$$f : G \rightarrow H$$

is _____

Select the correct option

☐ homomorphic

☐ isomorphic

☐ onto

☐ bijective

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Time Left 58 sec(s)

Quiz Start Time: 06:58 PM, 18 August 2018

Question # 2 of 10 (Start time: 06:58:47 PM, 18 August 2018)

Total Marks: 1

If N is a normal subgroup of $(G, \cdot)$, the set of cosets $G/N = \{Ng \mid g \in G\}$ forms a group $(G/N, \cdot)$, where the operation is defined by $(Ng1) \cdot (Ng2) = N(g1 \cdot g2)$. This group is called the quotient group or factor group of G by N .

Select the correct option

- | | |
|----------------------------------|-------|
| ☒ | True |
| ☐ | False |

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Time Left 28 sec(s)

Quiz Start Time: 06:58 PM, 18 August 2018

Question # 3 of 10 (Start time: 06:59:36 PM, 18 August 2018)

Total Marks: 1

The quotient group R/Z is isomorphic to the circle group

$$W = \{ e^{i\theta} \in \mathbb{C} \mid \theta \in \mathbb{R} \}.$$

Select the correct option

Reload Math Equations

☒	True
☐	False

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Time Left 74 sec(s)

Quiz Start Time: 06:58 PM, 18 August 2018

Question # 4 of 10 (Start time: 07:00:44 PM, 18 August 2018)

Total Marks: 1

The converse of Lagrange's theorem holds for abelian groups.

Select the correct option

- | | |
|----------------------------------|-------|
| ☒ | True |
| ☐ | False |

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Time Left 26 sec(s)

Quiz Start Time: 06:58 PM, 18 August 2018

Question # 5 of 10 (Start time: 07:01:04 PM, 18 August 2018)

Total Marks: 1

Let ϕ be a homomorphism of a group G into a group G' .
If H is a subgroup of G , then $\phi[H]$ is a subgroup of G' .

Select the correct option

Reload Math Equations

☒	True
☐	False

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Time Left 75 sec(s)
Quiz Start Time: 06:58 PM, 18 August 2018

Question # 6 of 10 (Start time: 07:02:16 PM, 18 August 2018) Total Marks: 1

Let $(G, \cdot)$ be a group with subgroup H . For $a, b \in G$,
we say $a \equiv b \pmod H$ if and only if $ab^{-1} \in H$.

Select the correct option

Reload Math Equations

☒	True
☐	False

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Time Left 81 sec(s)

Quiz Start Time: 06:58 PM, 18 August 2018

Question # 7 of 10 (Start time: 07:02:35 PM, 18 August 2018)

Total Marks: 1

The relation

$$a \equiv b$$

mod H is an equivalence relation on G.

Select the correct option

Reload Math Equations

☒	True
☐	False

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Time Left 73 sec(s)

Quiz Start Time: 06:58 PM, 18 August 2018

Question # 8 of 10 (Start time: 07:02:47 PM, 18 August 2018) Total Marks: 1

If N is a normal subgroup of a group G , the left cosets of N in G are the same as the right cosets of N in G .

Select the correct option

- | | |
|----------------------------------|-------|
| ☒ | True |
| ☐ | False |

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Time Left 71 sec(s)

Quiz Start Time: 06:58 PM, 18 August 2018

Question # 9 of 10 (Start time: 07:03:07 PM, 18 August 2018)

Total Marks: 1

If N is a normal subgroup of a group G , the left cosets of N in G are not the same as the right cosets of N in G

Select the correct option

- | | |
|----------------------------------|-------|
| ☒ | False |
| ☐ | True |

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Time Left 82 sec(s)

Quiz Start Time: 06:58 PM, 18 August 2018

Question # 10 of 10 (Start time: 07:03:31 PM, 18 August 2018)

Total Marks: 1

Let

$$\phi : G \rightarrow H$$

be a homomorphism, then $\phi(a^{-1}) = \phi(a)^{-1}$ for $a \in G$.

Select the correct option

Reload Math Equations

☒	True
☐	False

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Time Left 65 sec(s)

Quiz Start Time: 07:19 PM, 18 August 2018

Question # 3 of 10 (Start time: 07:19:31 PM, 18 August 2018)

Total Marks: 1

$(\mathbb{Z}_n, +)$ is a cyclic group with 1 as a generator.

Select the correct option

Reload Math Equations

☒	True
☐	False

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Time Left 83 sec(s)

Quiz Start Time: 07:19 PM, 18 August 2018

Question # 4 of 10 (Start time: 07:19:59 PM, 18 August 2018)

Total Marks: 1

The group G is a copy of the group H, if the function

$$f : G \rightarrow H$$

is _____

Select the correct option

Reload Math Equations

- | | |
|----------------------------------|-------------|
| ☐ | homomorphic |
| ☒ | isomorphic |
| ☐ | onto |
| ☐ | bijective |

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Time Left 77 sec(s)

Quiz Start Time: 07:19 PM, 18 August 2018

Question # 5 of 10 (Start time: 07:20:10 PM, 18 August 2018)

Total Marks: 1

Let H be a subgroup of a group G . Then $(aH)(bH) = a(bH)$ is not always true when G is non-abelian.

Select the correct option

- | | |
|----------------------------------|-------|
| ☐ | False |
| ☒ | True |

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Time Left 79 sec(s)

Quiz Start Time: 07:19 PM, 18 August 2018

Question # 6 of 10 (Start time: 07:20:27 PM, 18 August 2018) Total Marks: 1

Any subgroups of Abelian group G are not normal subgroups of G .

Select the correct option

- | | |
|----------------------------------|-------|
| ☒ | False |
| ☐ | True |

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Time Left 81 sec(s)

Quiz Start Time: 07:19 PM, 18 August 2018

Question # 7 of 10 (Start time: 07:20:46 PM, 18 August 2018)

Total Marks: 1

$Hg = gH$, for all $g \in G$, if and only if H is a normal subgroup of G .

Select the correct option

Reload Math Equations

☒	True
☐	False

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Time Left 79 sec(s)

Quiz Start Time: 07:19 PM, 18 August 2018

Question # 9 of 10 (Start time: 07:21:08 PM, 18 August 2018) Total Marks: 1

Let

$$\phi : G \rightarrow H$$

be an isomorphism, then if G is abelian, then H is not abelian.

Select the correct option

Reload Math Equations

☐	True
☒	False

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Time Left 79 sec(s)

Quiz Start Time: 07:19 PM, 18 August 2018

Question # 10 of 10 (Start time: 07:21:23 PM, 18 August 2018) Total Marks: 1

If N is a normal subgroup of a group G , the left cosets of N in G are the same as the right cosets of N in G .

Select the correct option

- | | |
|----------------------------------|-------|
| ☒ | True |
| ☐ | False |

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Time Left 80 sec(s)

Quiz Start Time: 07:31 PM, 18 August 2018

Question # 3 of 10 (Start time: 07:32:09 PM, 18 August 2018)

Total Marks: 1

Any subgroups of Abelian group G are not normal subgroups of G .

Select the correct option

- | | |
|----------------------------------|-------|
| ☒ | False |
| ☐ | True |

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Time Left 77 sec(s)

Quiz Start Time: 07:31 PM, 18 August 2018

Question # 4 of 10 (Start time: 07:32:26 PM, 18 August 2018)

Total Marks: 1

Let ϕ be a homomorphism of a group G into a group G' .
If K' is a subgroup of G' , then $\phi^{-1}[K']$ is a subgroup of G .

Select the correct option

Reload Math Equations

☒	True
☐	False

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Time Left 77 sec(s)

Quiz Start Time: 07:31 PM, 18 August 2018

Question # 6 of 10 (Start time: 07:32:51 PM, 18 August 2018) Total Marks: 1

Let

$$\phi : G \rightarrow H$$

be an isomorphism, then if G is abelian, then H is not abelian.

Select the correct option

Reload Math Equations

☐	True
☒	False

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Time Left 55 sec(s)

Quiz Start Time: 07:48 PM, 18 August 2018

Question # 1 of 10 (Start time: 07:48:23 PM, 18 August 2018) Total Marks: 1

Let

$$f: G \rightarrow H$$

be a homomorphism, then $f(e_G) = \text{---}$ for e_G and e_H are identities of G and H respectively.

Select the correct option

Reload Math Equations

☐	e_G
☒	e_H

Click to Save Answer & Move to Next Question

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Time Left 77 sec(s)

Quiz Start Time: 07:48 PM, 18 August 2018

Question # 7 of 10 (Start time: 07:50:22 PM, 18 August 2018)

Total Marks: 1

Let φ be a homomorphism function from group $(G, *)$ to group $(G', \cdot)$. Then, $(Ker \varphi, *)$ is a normal subgroup of $(G, *)$.

Select the correct option

Reload Math Equations

☒	True
☐	False

Click to Save Answer & Move to Next Question

QNo.8

Let $G = \langle a \mid a^n = e \rangle$ be a Finite Cyclic group of order n . Show that the mapping $\phi: a \rightarrow a^m$ is an automorphism of G iff $\gcd(m, n) = 1$.

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QNo.9

Classify the group $(\mathbb{Z}_4 \times \mathbb{Z}_2) / (\{0\} \times \mathbb{Z}_2)$ according to the Fundamental theorem of Finitely generated abelian groups.

QNo.1 Give an example of Trivial homomorphism. (2)

QNo.2 Define Simple group. (2)

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QNo.3 State Cauchy's theorem (2)

QNo.4 Write down all the generating sets of Group $\mathbb{Z}_6$.

QNo.5 Give an example of group action on set.

QNo.6 Show that the set of all ~~the~~ inner automorphisms of group G is a subgroup of $\text{Aut}(G)$.

QNo.7 Show that the set $\text{aut}(G)$ of all automorphisms of a group G is a group under composition of mapping, and $\text{Inn}(G) \triangleleft \text{Aut}(G)$.
Moreover, $G/Z(G) \cong \text{Inn}(G)$.

Note: Submit this sheet to Superintendent, before leaving the Examination Center.

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mth633
define

**stabilizer, commutator
and 2nd isomorphic
theorem.**

**$HVN=HN=NH$ wala ak
swal. factor wala ak
swal**

- ① Give Two examples of Normal groups 2 Marks
- ② Give an example of Trivial homomorphism
- ③ Give an example of Group action on a Set 3 Marks
- ④ Define Third theorem on Homomorphism
- ⑤ Prove for a prime 'p' every group G of order p^2 is Abelian 5 Marks
- ⑥ Show that $Z(G)$ is normal and Abelian Subgroup of G 5 Marks
- ⑦ Find Subgroup of $Z_4 \times Z_6$ which is generated by $(0, 2)$
- ⑧ Prove every cyclic group of a Subgroup is cyclic 3 Marks
- ⑨ Let $G = G_1 \times \dots \times G_i \times \dots \times G_n$ be a direct product, then prove the projection map $\pi_i: G \rightarrow G_i$ is Homomorphism 5 Marks.



Examples on Kernel of a Homomorphism

Example

Let $r \in \mathbb{Z}$ and let $\phi_r: \mathbb{Z} \rightarrow \mathbb{Z}$ be defined by $\phi_r(n) = rn$ for all $n \in \mathbb{Z}$. For all $m, n \in \mathbb{Z}$, we have $\phi_r(m+n) = r(m+n) = rm + rn = \phi_r(m) + \phi_r(n)$ so ϕ_r is a homomorphism.

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Examples on Kernel of a Homomorphism

MCQ

Note that ϕ_0 is the trivial homomorphism, ϕ_1 is the identity map, and ϕ_r maps $\mathbb{Z}$ onto $\mathbb{Z}$. For all other r in $\mathbb{Z}$, the map ϕ_r is not onto $\mathbb{Z}$.

$\text{Ker}(\phi_0) = \mathbb{Z}$
 $\text{Ker}(\phi_r) = \{0\}$ for $r \neq 0$

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Factor Group Computations

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Theorem

Let $G = H \times K$ be the direct product of groups H and K . Then $\bar{H} = \{(h, e) \mid h \in H\}$ is a normal subgroup of G . Also $G/\bar{H}$ is isomorphic to K in a natural way. Similarly, $G/\bar{K} \simeq H$ in a natural way.

Factor Group Computations

Theorem

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Factor Group Computations

Proof

Consider the map $\phi: H \times K \rightarrow K$ given by $\phi(h, k) = k$. The map ϕ is a homomorphism since $\phi(h_1 k_1, k_2 k_3) = \phi(h_1 k_1, k_2 k_3) = k_2 k_3 = \phi(h_1, k_2) \phi(k_1, k_3)$. Because $\ker(\phi) = \bar{H}$, we see that $\bar{H}$ is a normal subgroup of $H \times K$. Because ϕ is onto K , Fundamental Theorem of Homomorphisms tells us that $(H \times K)/\bar{H} \simeq K$.

Group Theory

Factor Group Computations

Factor Group Computations

Example

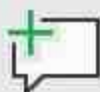
Let us compute the factor group $(\mathbb{Z} \times \mathbb{Z})/(\mathbb{Z} \times \{0\})$. We see that $\mathbb{Z} \times \{0\}$ is a normal subgroup of $\mathbb{Z} \times \mathbb{Z}$. There is a natural map $\phi: \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}$ defined by $\phi(a, b) = b$. The map ϕ is a homomorphism since $\phi(a_1 b_1, c_1 d_1) = \phi(a_1 b_1, c_1 d_1) = c_1 d_1 = \phi(a_1, c_1) \phi(b_1, d_1)$. Because $\ker(\phi) = \mathbb{Z} \times \{0\}$, we see that $\mathbb{Z} \times \mathbb{Z}/(\mathbb{Z} \times \{0\}) \simeq \mathbb{Z}$. This is what we wanted to show.

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13

14

15



The Commutator Subgroup

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Thus the commutator subgroup C of S_3 contains A_3 . Since A_3 is a normal subgroup of S_3 and S_3/A_3 is abelian, above theorem shows that $C=A_3$.

Group Theory	The Commutator Subgroup	The Commutator Subgroup	The Commutator Subgroup

1

2

3

4



Isomorphism Theorems

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If H and N are subgroups of a group G , then we let $HN = \{hn \mid h \in H, n \in N\}$. We define the **join** $H \vee N$ of H and N as the intersection of all subgroups of G that contain HN ; thus $H \vee N$ is the smallest subgroup of G containing HN .

Isomorphism Theorems	Isomorphism Theorems	Isomorphism Theorems	Isomorphism Theorems
If H and N are subgroups of a group G , then we let $HN = \{hn \mid h \in H, n \in N\}$. We define the join $H \vee N$ of H and N as the intersection of all subgroups of G that contain HN ; thus $H \vee N$ is the smallest subgroup of G containing HN .	If H and N are subgroups of a group G , then we let $HN = \{hn \mid h \in H, n \in N\}$. We define the join $H \vee N$ of H and N as the intersection of all subgroups of G that contain HN ; thus $H \vee N$ is the smallest subgroup of G containing HN .	Lemma If H is a normal subgroup of G , and if N is any subgroup of G , then HN is a normal subgroup of G . Furthermore, if N is also normal in G , then HN is normal in G .	Proof We show that HN is a subgroup of G , then what $H \vee N$ is follows at once. Let $h_1, h_2 \in H$ and $n_1, n_2 \in N$. Since H is a normal subgroup, we have $h_1 n_1 h_1^{-1} \in N$. Then $h_1 n_1 h_1^{-1} n_2^{-1} \in N$, so $h_1 n_1 h_1^{-1} n_2^{-1} n_2 \in N$. Clearly $n_2 \in N$, so $h_1 n_1 h_1^{-1} n_2 \in N$. Also, $h_1 n_1 h_1^{-1} \in H$, so $h_1 n_1 h_1^{-1} n_2 \in HN$. Thus HN is a normal subgroup. Thus $H \vee N = HN$, as $HN \subseteq H \vee N$.



Generating Sets

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Example

The Klein 4-group $V = \{e, a, b, c\}$ is generated by $\{a,b\}$ since $ab=c$.

It is also generated by $\{a,c\}$, $\{b,c\}$, and $\{a,b,c\}$.

If a group G is generated by a subset S , then every subset of G containing S generates G .

Generating Sets	Group Theory	Generating Sets	Generating Sets
Example: The Klein 4-group $V = \{e, a, b, c\}$ is generated by $\{a,b\}$ since $ab=c$. It is also generated by $\{a,c\}$, $\{b,c\}$, and $\{a,b,c\}$. If a group G is generated by a subset S, then every subset of G containing S generates G.	Generating Sets	Example: The group $\mathbb{Z}$ is generated by $\{0\}$ and $\{1\}$. It is also generated by $\{2,3\}$ since $3-2=1$, so that any subgroup containing 2 and 3 must contain 1 and must therefore be $\mathbb{Z}$.	$\mathbb{Z}$ is also generated by $\{2,3\}$, $\{3,4\}$, $\{3,5\}$, and $\{3,6\}$. But it is not generated by $\{2, 4\}$ since $\langle 2 \rangle = \{0, 2, 4\}$ contains 2 and 4.



Simple Groups

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Example

On the other hand, the group $G = \mathbb{Z}/12\mathbb{Z}$ is not simple.

The set $H = \{0, 4, 8\}$ of congruence classes of 0, 4, and 8 modulo 12 is a subgroup of order 3, and it is a normal subgroup since any subgroup of an abelian group is normal.

Simple Groups	Simple Groups	Simple Groups	Group Theory
Example On the other hand, the group $G = \mathbb{Z}/12\mathbb{Z}$ is not simple. The set $H = \{0, 4, 8\}$ of congruence classes of 0, 4, and 8 modulo 12 is a subgroup of order 3, and it is a normal subgroup since any subgroup of an abelian group is normal.	Example The additive group $\mathbb{Z}$ of integers is not simple. The set of even integers $2\mathbb{Z}$ is a non-trivial proper normal subgroup.	Theorem The alternating group A_n is simple for $n \geq 5$.	Simple Groups

