DATA for QUIZ#2---MTH633---MODULE # 77 to 97

1.	A translation of the plane R^2 in the direction of the vector (a, b) is a function $f: R^2 \to R^2$ defined by $f(x, y) = (x + a, y + b)$.
2.	The composition of this translation with a translation g in the direction of (c, d) is the function $f\,g: R^2 \to R^2$, where $f\,g(x, y) = (x + c + a, y + d + b)$ This is a translation in the direction of $(c + a, d + b)$
3.	The set of all translations in R^2 forms an abelian group, under composition.
4.	A translation of the plane R^2 in the direction of the vector (0, 0) is an identity function $1_{R^2}: R^2 \to R^2$ defined by $1_{R^2}(x, y) = (x+0, y+0) = (x, y)$.
5.	The inverse of the translation of the plane R^2 in the direction of the vector (a, b) is an inverse function $f^{-1}: R^2 \to R^2$ defined by $f^{-1}(x, y) = (x - a, y - b)$ such that $f\,f^{-1}(x, y) = (x, y) = f^{-1}f(x, y)$
6.	The inverse of the translation in the direction (a, b) is the translation in the opposite direction $(-a, -b)$.
7.	An isomorphism $\phi: G \to G'$, if one exists, is an example of structure-relating map.
8.	If$(G, \cdot)$ and$(H, *)$ are two groups, the function $f: G \to H$ is called a group homomorphism if $f(a \cdot b) = f(a) * f(b)$ for all $a, b \in G$.
9.	We often use the notation $f: (G, \cdot) \to (H, *)$ for such a homorphism. Many authors use morphism instead of homomorphism.
10.	A group isomorphism is a bijective group homomorphism, If there is an isomorphism between the groups $(G, \cdot)$ and $(H, *)$, we say that $(G, \cdot)$ and $(H, *)$ are isomorphic and write $(G, \cdot) \cong (H, *)$
11.	Let $\phi: G \to G'$ be a group homomorphism of G onto G' then if G is abelian, then G' must be abelian.
12.	The function $f: Z \to Z_n$, defined by $f(x) = [x]$ is the group homomorphism (EXAMPLE P#14)
13.	The function $f: R \to R^+$, defined by $f(x) = e^x$ is a homomorphism, for if $x, y \in R$, then $f(x + y) = e^{x+y} = e^x e^y = f(x)\,f(y)$. (EXAMPLE P#15)
14.	Now f is an isomorphism, for its inverse function $g: R^+ \to R$ is ln x. The additive group R is isomorphic to the multiplicative group R^+. The inverse function g is also an isomorphism.
15.	Let S_n be the symmetric group on n letters and let $\phi: S_n \to \mathbb{Z}_2$be defined by $\phi(\sigma) = 0$ if σ is an even permutation, $\phi(\sigma) = 1$ if σ is an odd permutation.
16.	If σ and μ can both be written as a product of an odd number of transpositions then $\sigma\mu$ can be written as the product of an even number of transpositions.
17.	Let $\phi: G \to H$ be a group morphism and let e_G and e_H be the identities of G and H, respectively. Then (i) $\phi(e_G) = e_H$
18.	Let $\phi: G \to H$ be a group morphism and let e_G and e_H be the identities of G and H, respectively. Then (ii) $\phi(a^{-1}) = \phi(a)^{-1}$for all $a \in G$.
19.	Let ϕ be a mapping of a set X into a set Y and let $A \subseteq X$ and $B \subseteq Y$. The image $\phi[A]$ of A in Y under ϕ is $\{\phi(a) \mid a \in A\}$. The set $\phi[X]$ is the range of ϕ. The inverse image $\phi^{-1}[B]$ of B in X is $\{x \in X \mid \phi(x) \in B\}$.
20.	Let ϕ be a homomorphism of a group G into a group G'. If H is a subgroup of G then $\phi[H]$ is a subgroup of G'. If K' is a subgroup of G' then $\phi^{-1}[K']$ is a subgroup of G.
21.	If N is a normal subgroup of a group G, the left cosets of N in G are the same as the right cosets of N in G.

22.	Let h be a homomorphism from a group G into a group G'. Let K be the kernel of h. Then $a K = \{x \text{ in } G \mid h(x) = h(a)\} = h^{-1}[\{h(a)\}]$ and also $K a = \{x \text{ in } G \mid h(x) = h(a)\} = h^{-1}[\{h(a)\}]$
23.	When h is a homomorphism of groups then the cosets(left or right) of the kernel of h are the equivalence classes of this equivalence relation.
24.	If $h: X \to Y$ is any map of sets Then h defines an equivalence relation $\sim_h$ on X by: $x \sim_h y \Leftrightarrow h(x) = h(y)$
25.	If $\phi:G\to G'$ is a group morphism, then kernel of ϕ is defined to be the set of elements of G that are mapped by f to the identity of G'. That is $\text{Ker } f=\{g\in G \mid f(g)=e'\}$
26.	Let $\phi:G\to G'$ be a group morphism Then ϕ is injective if and only if $\text{Ker } \phi=\{e\}$
27.	To Show $\phi:G \to G'$ is an Isomorphism Step 1 Show ϕ is a homomorphism. Step 2 Show $\text{Ker}(\phi)= \{e\}$. Step 3 Show ϕ maps G onto G'.
28.	Let G be a group with subgroup H. The *right cosets* of H in G are equivalence classes under the relation $a\equiv b \bmod H$, defined by $ab^{-1}\in H$
29.	Let G be a group ,we can define the relation L on G so that aLb if and only if $b^{-1}a\in H$. This relation L is an equivalence relation and the equivalence class containing a is the *left coset* $aH=\{ah \mid h\in H\}$
30.	The right cosets of $H=A_3$ in S_3 are $H=\{(1),(123),(132)\};H(12)=\{(12),(13),(23)\}$ The left cosets of $H=A_3$ in S_3 are $H=\{(1),(123),(132\};(12)H=\{(12),(23),(13)\}$
31.	The right cosets of $K=\{(1),(12)\}$ in S_3 are $K=\{(1),(12)\};K(13)=\{(13),(132)\};K(23)=\{(23),(123)\}$ The left cosets of $K=\{(1),(12)\}$ in S_3 are $K=\{(1),(12)\};(23)K=\{(23),(132)\};(13)K=\{(13),(123)\}$
32.	A subgroup H of a group G is called a *normal subgroup* of G if $g^{-1}hg \in H$ for all $g \in G$ and $h \in H$.
33.	Also $Hg=gH$ for all $g\in G$ if and only if H is a normal sub group of G.
34.	If N is a normal subgroup of $(G, \cdot)$, the set of cosets $G/N = \{Ng \mid g\in G\}$ forms a group $(G/N, \cdot)$, where the operation is defined by $(Ng_1) \cdot (Ng_2) = N(g_1 \cdot g_2)$ then • This group is called the quotient group or factor group of G by N. • Multiplication of cosets is well defined • The operation is associative • The identity is $Ne=N$ • The inverse of Ng is Ng^{-1} • $Ng^{-1}\cdot Ng=N$ Hence $(G/N,\cdot)$ is a group.
35.	The quotient group $\mathbb{R}/\mathbb{Z}$ is isomorphic to the circle group $W = \{e^{i\theta} \in \mathbb{C} \mid \theta \in \mathbb{R}\}$ i.e $\mathbb{R}/\mathbb{Z} \cong W$.
36.	The morphism $f : \mathbb{R} \to W$ is clearly surjective and its kernel is $\{x \in \mathbb{R} \mid e^{2\pi i x} = 1\} = \mathbb{Z}$
37.	The relation $a \equiv b \bmod H$ is an equivalence relation on G.

38.	Let (G,·) be a group with subgroup H. For a,b∈G, we say that a is ***congruent to b modulo H*** and write **a≡b mod H** if and only if ab^{-1}∈H
39.	($\mathbb{Z}_n$,+) is the quotient group of ($\mathbb{Z}$,+) by the subgroup n$\mathbb{Z}$={n|z∈$\mathbb{Z}$}. • If ($\mathbb{Z}$,+) is abelian then every subgroup is normal • The set $n\mathbb{Z}$ can be verified to be a subgroup • The relationship a≡b mod $n\mathbb{Z}$ is equivalent to a−b∈ $\mathbb{Z}n$ and to n|a−b • a≡b mod $n\mathbb{Z}$ is the same relation as a≡b mod n • Therefore $\mathbb{Z}_n$ is the quotient group $\mathbb{Z}/n\mathbb{Z}$ where the operation on congruence classes is defined by [a]+[b]=[a+b] • ($\mathbb{Z}_n$,+) is a cyclic group with 1 as a generator.
40.	Let K be the kernel of the group morphism f :G → H Then G/K is isomorphic to the image of f and the isomorphism ψ: G/K → I m f is defined by ψ(Kg) = f(g). • This result is also known as the first isomorphism theorem. • The function ψ is a morphism because ψ(Kg1Kg2) = ψ(Kg1)ψ(Kg2). • If ψ(Kg) = eH then f(g) = eH and g ∈K. Hence the only element in the kernel of ψ is the identity coset K and ψ is injective.
41.	The equivalence class containing a can be written in the form Ha = {ha|h∈H} It is called a right coset of H in G. The element a is called a representative of the coset Ha.
42.	Let φ be a homomorphism function from group (G, *) to group (G', .) Then (Kerφ,*) is a normal subgroup of (G,*)
43.	A subgroup H of a group is a normal subgroup if gH=Hg for $\forall g$ ∈G.
44.	Any subgroups of Abelian group are normal subgroups.
45.	If Hg=gH then there exists h'∈H such that hg=gh' for $\forall h \in H$.
46.	Hg=gH, it does not imply hg=gh.
47.	Let H be a subgroup of a group G. When is (a H) (b H) = a b H? • This is true for abelian groups but not always when G is non abelian. • Consider S3: Let H = {ρ0, μ1}. The left cosets are {ρ0, μ1}, {ρ1, μ3}, {ρ2, μ2}. • If we multiply the first two together, then {ρ0, μ1}, {ρ1, μ3} = {ρ0ρ1, ρ0μ3,μ 1ρ1, μ 1μ3} = {ρ1, μ3,μ2, ρ 2} • This has four distinct elements, not two!
48.	Let (H,*) be a normal subgroup of the group (G,*). (G/H,⊗)is called quotient group, where the operation ⊗ is defined on G/H by $Hg_1 \otimes Hg_2$= H(g1*g2). If G is a finite group, then G/H is also a finite group and |G/H|=|G|/|H|.
49.	The product of two sets is define as follow SS' = {xx'|x∈S and x'∈S}
50.	{aH|a∈G, H is normal} is a group denote by G/H and called it factor groups of G.
51.	A mapping f: G→G/H is a homomorphism and call it canonical homomorphism.
52.	Consider S3: Let H = {ρ0, ρ1, ρ2}. The left cosets are {ρ0, ρ1, ρ2}, {μ1, μ2, μ3} The cosets of {ρ0, ρ1, ρ2} with this binary operation form a group isomorphic to $\mathbb{Z}_2$.
53.	There is a natural map from S_3 to {{ρ0, ρ1, ρ2}, {μ1, μ2, μ3}} that takes any element to the coset that contains it. This gives a homomorphism called the canonical homomorphism.
54.	Let H be a subgroup of a group G Then H is normal if and only if (a H)(b H) = (a b) H, for all a, b in G
55.	Lagrange's Theorem shows that if there is a subgroup H of a finite group G, then the order of H divides the order of G.
56.	The converse of Lagrange's theorem holds for abelian groups.